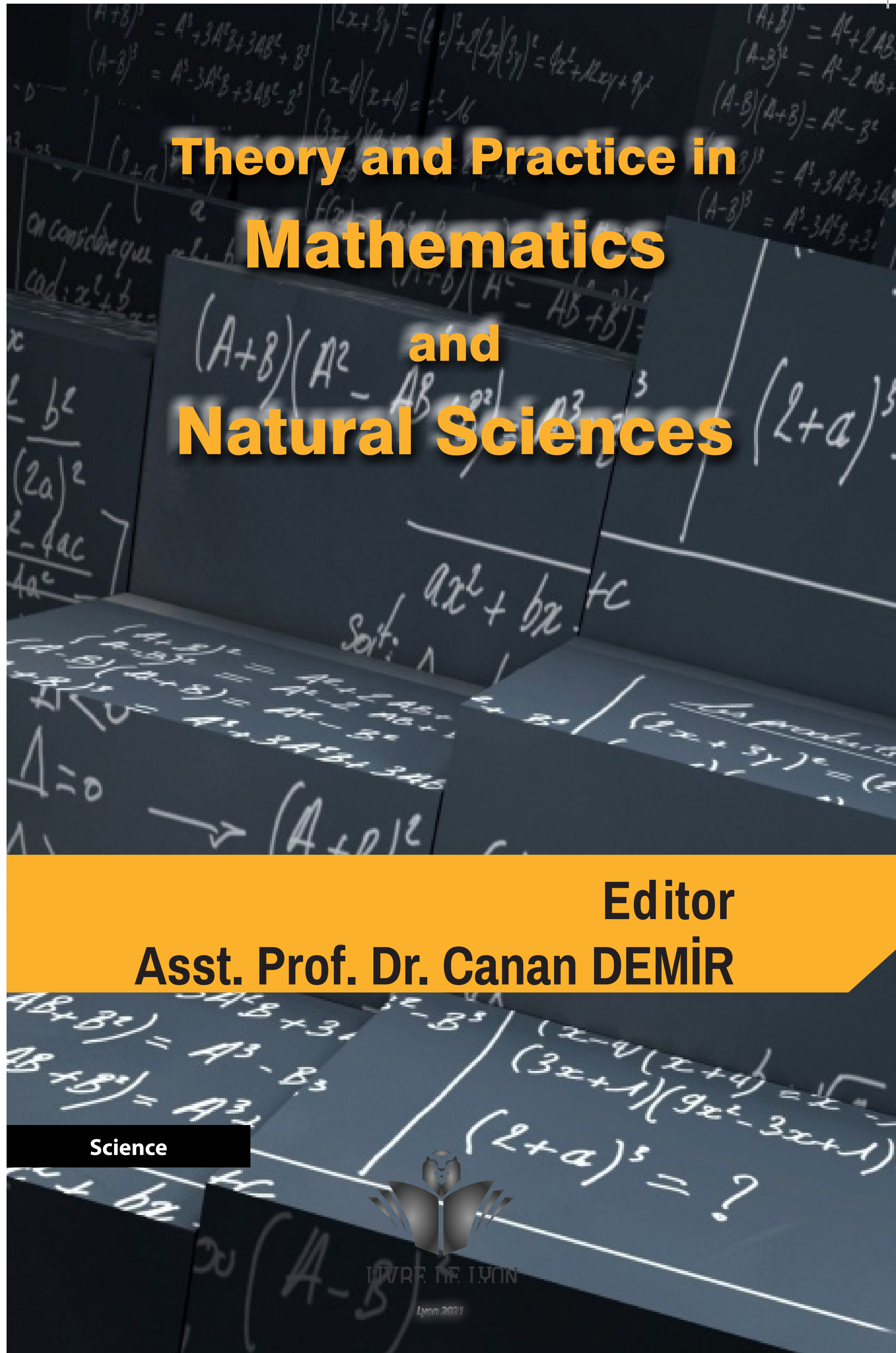
Theory and Practice in Mathematics and Natural Sciences



Editor

Asst. Prof. Dr. Canan DEMİR

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PREFACE

Dear readers;

In this book, which is about Theory and Practice in mathematics and Natural Sciences, there are 7 chapters in total, including valuable studies and written according to certain criteria. It is our greatest desire that this book will be an important resource for both students, academics and readers working or studying in the field of Mathematics and Natural Sciences. I would like to thank the publishing house and the authors who contributed to the writing, shape, design and preparation of this edition.

Regards,

Asst. Prof. Dr. Canan Demir, 2021

CHAPTER I

PISA MATHEMATICAL LITERACY

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1. Introduction

Among the themes at the centre of mathematics are the relationships between mathematics and real life (Blum, 2002). Because the concepts in mathematics have connections and applications that are not far from human life. These applications are encountered not only in mathematics but also in the applications of other branches of science that try to explain life. Mathematics teaches us to solve problems within the framework of a scientific process. In this respect, mathematics is also among the most effective tools to help solve complex problems such as population growth, flood, hunger, and epidemics (Banu, 1991). Mathematics is a subject in school life, as well as in real life, at all levels from kindergarten, primary school to higher education.

It has become the focus of politicians and education systems around the world to equip individuals with the knowledge and abilities to maximize their current potential to contribute to an increasingly global, inter-integrated world and transform their knowledge and abilities into a better life (OECD, 2019a). The current and future positions of countries will vary according to the quality of education they have. It is essential to have a good education system in order to raise future generations and for countries to have an effective and strong place among nations. It is of great importance to evaluate the knowledge and skills of students, to evaluate the education systems of countries and to compare them with other countries in raising students who can keep up with the needs of the age thanks to a quality education and this education. Internationally conducted TIMSS (Trends in International Mathematics and Science Study) and PISA (Program for International Student Assessment) applications provide countries with data on education systems and enable countries to compare their education systems at international level. In the following sections, information about PISA, which is one of the most effective international assessment programs, and mathematical literacy, which is seen as one of the most important competencies that PISA aims to measure, will be presented.

2. PISA Framework

In response to the need to present an internationally comparable situation regarding student performance in line with the idea of "what is important for individuals to know and be able to do", PISA, called the “Program for International Student Assessment” organized by the Organization for Economic Co-Operation and Development-OECD, has over the past years improved the equity, quality and efficiency of school systems. It has become the world's leading benchmark for evaluation (OECD, 2016). 15-year-old students who have completed the compulsory education of many OECD and non-OECD member countries participate in this research, which started with the participation of 32 OECD member and non-member countries and has continued every three years since 2000 (Participating countries according to the years of implementation can be found in the table in Appendix 1.). As of 4 May 2020, 85 countries have applied to participate in the study, which will be carried out in 2022. In PISA applications, it is evaluated to what extent 15-year-old students in the participating countries, which constitute approximately 90% of the world economy, have the necessary knowledge and skills (MoNE, 2011). In the light of this information, it offers us ideas for the best education policies and practices, in line with the report presented by PISA applications on education levels all over the world every three years.

Prior to PISA, the OECD's main basis for comparison of educational outcomes until the late 1990s was education time, which did not offer any insight into what individuals could do with what they learned during school life (OECD, 2019a). With PISA, this situation has changed and not only how much students remember their knowledge, but also their knowledge and experiences have been examined, their knowledge and skills for real-life situations have been evaluated (OECD, 2005). In other words, PISA focused on both the school life and the life outside of the school rather than the education period of the students (Stacey & Turner, 2015). The basic idea behind the applications is to directly test students' knowledge and skills in line with an internationally accepted assessment (OECD, 2019b). In this direction, PISA applications are thought to be an important measurement tool in terms of determining what level of skills students can use in real life problems and at what level of problems they have difficulty.

Students are evaluated in reading, mathematics and science skills every three years in the PISA exams. With this evaluation, the aim is not to evaluate educators or policies within the framework of accountability, but to prepare the educational structures and systems for the future by focusing the perspectives of policy makers and education programs on future students, teachers, schools and programs. The PISA application, which was carried out for the first time in 2000, focuses on different subject areas in each application period. In Figure 1, the cycle including subject areas and weighted areas in PISA applications by years is presented:

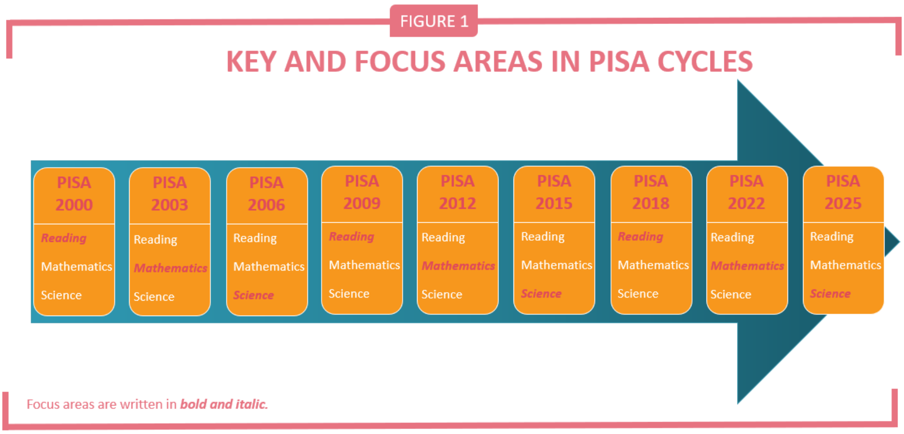


Figure 1. Key and Focus Areas in PISA Cycles

Financial literacy performances of students were also evaluated in 2012 for the first time in PISA, which evaluate the performances of 15-year-old students in the participating countries in the fields of reading skills, mathematics and science (OECD, 2014). In a small part of the evaluation in the PISA 2012, the problem-solving performances of the students were also emphasized. In the PISA conducted in 2015, the application was carried out in a way that the majority of the working area was science literacy. In the 2018 PISA, reading skills were the dominant area. In the application in 2015 and 2018, the financial literacy performances of the students were also examined, as in 2012. Furthermore, the performance of the students in the field of problem solving was also evaluated in the application in 2015, as in 2012. While the next application of PISA is planned to be in 2021, it will be carried out in 2022 due to the pandemic process. The focus of the application to be made in 2022 will be on mathematical literacy.

In PISA applications, evaluations are made in terms of gender and socio-economic levels as well as student performances. In the 2018 application, besides student applications, evaluations for well-being, ICT family, school, teacher, parents were included. While evaluating students' knowledge, an evaluation framework is presented about the factors that affect students' development at home and at school, and how these factors interact with students' knowledge and skills (OECD, 2019b). Through these applied questionnaires, perspectives on many factors such as school leadership, student profile, weight in the curriculum, extracurricular activities, socio-economic infrastructure of the school, the region and size of the school, students' learning styles, mathematics, science and reading skills, parents' thoughts about the student and the school are put forward (Yavuz, 2014). One of the most important information revealed by PISA applications is that it contains enlightening information about countries that are successful in applications and meet certain standards (OECD, 2019a). In this direction, many countries have started to take PISA applications as reference while developing their education policies (Grek, 2009; Breakspear, 2012). The results obtained from the PISA applications provide the opportunity to evaluate the education strategies of the participating countries, as well as the opportunity to organize and develop future education policies in this direction by examining the education practices in successful countries.

The fact that one of the main themes in PISA applications is mathematical literacy is another reason for drawing attention to PISA applications (Altun & Bozkurt, 2017). In a world where those who show not only what they know but also what they can do with what they know are increasingly rewarded, students need to infer from what they know and apply these inferences to new situations in order to be successful. Accordingly, mathematical literacy, which focuses not only on school life but also on life after school, focuses not only on what students know, but also on what they can do with what they know (Dossey, McCrone, Turner, & Lindquist, 2008). Today, countries need to prepare their education policies and programs for the future in order to achieve success. In this respect, as long as mathematical literacy focuses on the future, together with PISA applications, it will remain up-to-date. In the following sections, information on mathematical literacy and classification of mathematical literacy problems will be presented within the framework of PISA.

3- Mathematical Literacy

In recent years, mathematics has begun to be seen as knowledge and skills revealed through the problem solving process, which is seen as a model of real life rather than an abstract concept and the relations between these concepts (De Corte, 2004). This approach has led to an understanding that can use mathematics in real life, solve problems by using mathematical thinking skills in the face of problems and apply what they have learned in daily life. Such an approach emerges as mathematical literacy.

While the first statements about the concept of mathematical literacy date back to 1944, it is stated that the first attempt to define it was made in 1999 within the framework of the OECD-PISA framework (Niss, 2015, p. 410). Although mathematical literacy has strong connections with concepts such as "numeracy" and "quantitative literacy" in the literature in terms of the meaning it contains, it is seen that more emphasis is placed on mathematical literacy among these concepts (Turner, 2012). Until the definition of mathematical literacy by PISA, there was no clear assessment of what the concepts of "numeracy" and "quantitative literacy" meant, and a universal definition of these concepts or the determination of their components. With the definition of the concept of "mathematical literacy" and the explanation of its components in PISA reports, the concept of literacy in modern societies has entered the educational dictionary and started to be mentioned with PISA (Stacey & Turner, 2015).

The first definition of mathematical literacy within the framework of PISA is examined as follows:

> *"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen"* (OECD 1999, p. 41).

Turner (2012) states that this definition aims to create a broad perspective on mathematics that includes a wide participation and use of mathematics in all aspects of life. OECD has organized this definition within the framework of PISA 2006 application as follows:

> *"Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen"* (OECD,2006, p. 72).

Within the framework of definitions, mathematical literacy is not only limited to mastering the subjects in the school curriculum, but also describes the capacity to use mathematical knowledge and skills (Kramarski & Mizrachi, 2004) and requires high-level thinking skills for mathematics (Meaney, 2007). Similarly, Steen, Turner, and Burkhardt (2007, p. 286) emphasize three important points in mathematical literacy in line with the OECD's definition. The first is that mathematical literacy is more than numeracy or basic skills, and the second is that it requires something quite different from traditional school mathematics for mathematical literacy. The third important point is that it cannot be separated from mathematical literacy contexts.

The definition of mathematical literacy used in previous years in the PISA 2012 Framework (OECD 2013a) has been revised again, but there has been no change in the basic structure of the literacy framework. The current definition made in the 2012 framework aimed to clarify the ideas that support mathematical literacy, thus making it more transparent and operational, and to more clearly define the fundamental and growing role that mathematics plays in modern society (Stacey & Turner, 2015). Definition updated in the framework of PISA 2012:

> *"Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens"* (OECD, 2013b, p. 25).

The updated definition emphasizes the capacity of individuals within the context of mathematical literacy. Formulating real-life situations as mathematical models, obtaining results by working on the formulated model, and interpreting and evaluating this result are suggested as basic processes. In the continuation of the definition, it is stated that mathematical literacy includes all aspects of mathematics, whether it is mathematical reasoning or the method of concepts and techniques. In this respect, it is possible to say that mathematical literacy enables people to be aware of the role that mathematics plays in the modern world, to make applications related to daily life, to develop skills, to interpret in numerical and spatial thinking, to feel confident, to critical analysis and problem solving in daily life situations (Özgen & Bindak, 2008). When we evaluate the definition made from another perspective, if the student activates his mathematical capacity and perceptions when he encounters a problem, and makes use of his mathematical knowledge and skills in solving this problem, this student can be considered as mathematically literate (Altun, 2014). It includes high-level skills such as solving problems given in the context of real life, using mathematical knowledge and skills, internalizing and interpreting mathematics (MoNE, 2015). In this respect, mathematical literacy is not about studying at higher levels of formal mathematics, but about making mathematics relevant and empowering for everyone (McCrone & Dossey, 2007). In addition, the definition put forward in the framework of PISA 2012 includes the definition of which competences mathematical literacy consists of more precisely (Stacey, 2015).

Within the framework of PISA 2012, a model for mathematical literacy was presented as follows:

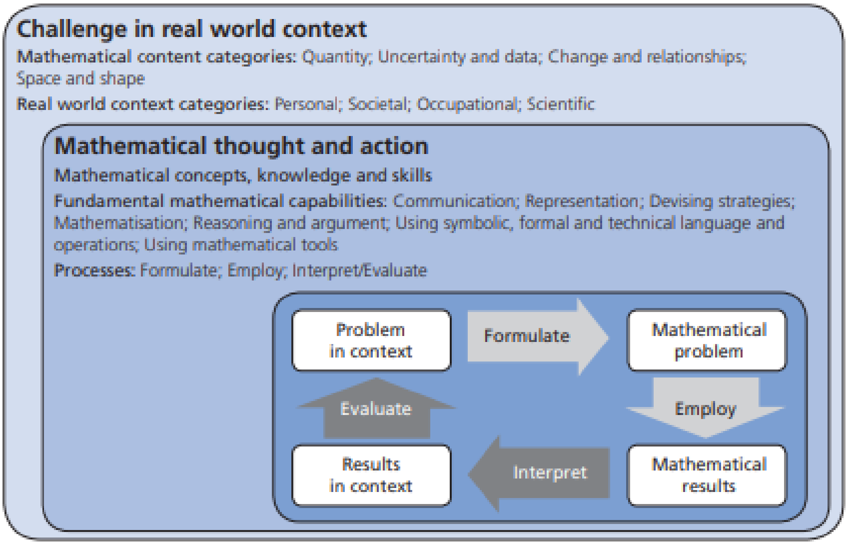


Figure 2. A Model of Mathematical Literacy in PISA 2012 (OECD, 2013b, p.26)

The model in Figure 2 presents a perspective on the main themes of mathematical literacy and how these themes are related to each other. When we focus on the model in general, it is shown that mathematical literacy is necessary to overcome a real-world challenge. This difficulty arises in two category components: the mathematical content and the real-world context of the nature of the situation. The middle frame in the model, otherwise, refers to the mathematical thinking, knowledge and skills that should be used in solving the difficulties encountered. The processes specified in the diagram are formulated according to the mathematical modelling processes of the problem taken from real life, and importance is given to the use of mathematics for the solution of the formulated problem and the interpretation of the results obtained (MoNE, 2011). It also shows the processes that the problem solver goes through while solving a problem (Stacey & Turner, 2015). As it is revealed in the diagram given, an individual with mathematical literacy can formulate a problem he encounters in the context of daily life and transfer it to the mathematical world, he can solve this problem that he expresses mathematically by using his knowledge and skills, and by making comments and evaluations on the results, he can solve other problems he may encounter. In addition, based on this schema presented in the PISA 2012 report, it can be said that an individual with mathematical literacy can have the ability to deal with, understand and diagnose mathematics, and in this direction, he or she can make well-founded judgments about the function of mathematics in his life and in his general life in the future (OECD, 2014). Dossey et al. (2008) state that mathematical literacy defined in the PISA program is defined as the capacity of individuals to understand and define the role played by mathematics in the world.

PISA will focus on mathematical literacy in 2022. In this direction, the PISA 2022 Framework was shared and an up-to-date definition of mathematical literacy was made:

> "*Mathematical literacy is an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of realworld contexts. It includes concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to know the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective 21st century citizens"* (OECD, 2018, p. 7).

Compared to the PISA 2003 and PISA 2012 frameworks, it can be said that the PISA 2022 framework appreciates and preserves the core ideas of mathematical literacy developed, the real-world context of mathematical literacy problems is made more specific, the importance of mathematical reasoning becomes more apparent, and concepts related to the 21st century citizens is focused on. The definition of mathematical literacy not only focuses on the use of mathematics to solve real-world problems, but also considers mathematical reasoning as a core aspect of mathematical literacy. In this respect, the centrality of mathematical reasoning is emphasized both for the problem-solving cycle and for mathematical literacy in general (OECD, 2018). The model introduced in PISA 2012 (Figure 2) within the framework of PISA 2022 (Figure 3) has also been updated:

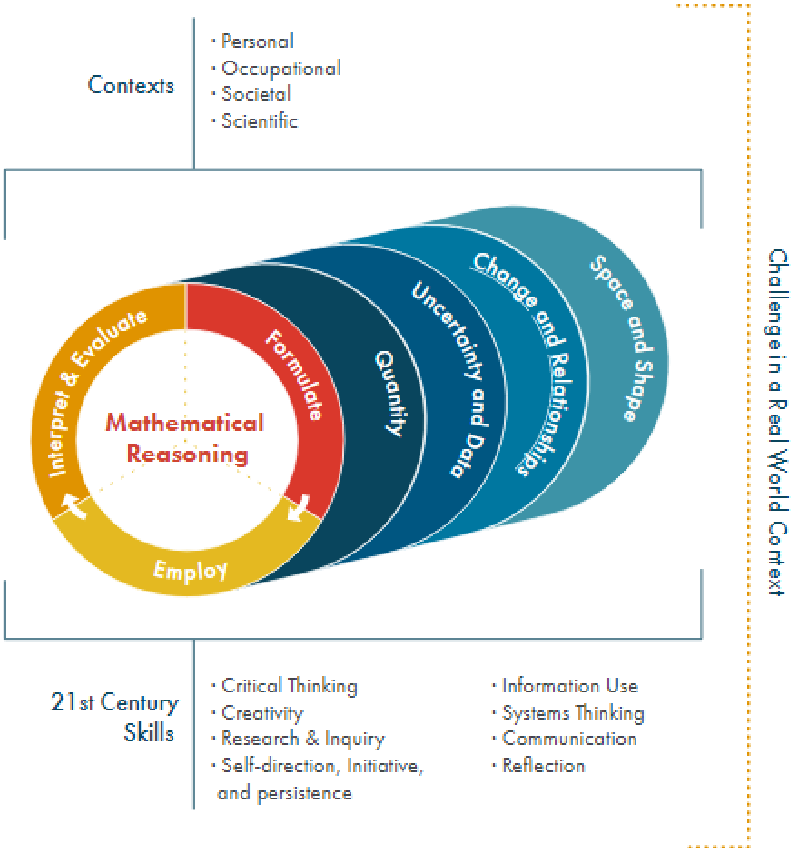


Figure 3. Model of Mathematical Literacy in PISA 2022 (OECD, 2018)

While the main themes revealed in PISA 2012 are included in the model cycle within the framework of PISA 2022, it is noteworthy that mathematical reasoning is at the centre of the model. In the framework of PISA 2012, the relationship between the mathematical process in the model for solving mathematical literacy problems and mathematical reasoning is shown. The relationship between mathematical reasoning and process skills is explained as follows:

> "*In order for students to be mathematically literate they must be able, first to use their mathematics content knowledge to recognise the mathematical nature of a situation (problem) especially those situations encountered in the real world and then to formulate it in mathematical terms. This transformation – from an ambiguous, messy, real-world situation to a well-defined mathematics problem – requires mathematical reasoning. Once the transformation is*

successfully made, the resulting mathematical problem needs to be solved using the mathematics concepts, algorithms and procedures taught in schools. However, it may require the making of strategic decisions about the selection of those tools and the order of their application – this is also a manifestation of mathematical reasoning" (OECD, 2018, p.8).

Another notable concept in this model is the 21st century skills, which has recently attracted attention in the world. 21st century skills are included in PISA applications for the first time. When the model is examined, it is seen that a framework is created for 21st century skills mathematical literacy, just like contexts. 21st century skills will contribute significantly to the creation of mathematical literacy problems, and mathematical literacy will contribute to the development of 21st century skills.

In general, within the framework of PISA applications, it is seen that mathematical literacy problems are handled in line with the mathematical content (subject area) and the real world. Since 2012, it has been classified as skills for problem solving processes. From the PISA 2003 to the PISA 2012, the problems were also categorized as competencies and competency cluster. The classification made in line with competencies and competency cluster was not used in the 2012 practice because it did not fully meet the expectations in PISA evaluations (Altun & Bozkurt, 2017). In the following section, PISA mathematical literacy problems will be discussed from three different aspects: mathematical *content*, real world *context* and mathematical *process skills*.

4. Classification of PISA Mathematical Literacy Problems

PISA mathematical literacy problems are classified as follows:

a) Mathematical *content*,
b) Real world *context*,
c) Mathematical process skills
d) Competencies[1]
e) Competency cluster[1]

The mathematical content includes the subject areas from which the problems are selected. Context, on the other hand, is expressed as "the world of the individual in which the problems take place" (MoNE, 2011, p.25). "Mathematical processes explain what individuals do to relate the

[1] After the PISA 2012 application, mathematical literacy problems were not classified in this framework.

context of the problem with mathematics and solve it" (MoNE, 2011, p.14).

Competencies appear as cognitive processes that become active in the mathematical solution phase to solve the problem presented in a context (Saenz, 2009). Competencies are activated in different ways and come into play at different levels as students encounter problems (Dossey et al., 2008). PISA defined these competencies in 6 categories: reasoning and argumentation; communication; modelling; problem solving; representation; and symbols and symbolic expressions. It may be a problem to include these competencies in each of the problems or to include each competency at the same level in a problem (Altun & Bozkurt, 2017). Similarly, Stacey and Turner (2015) stated that mathematical competencies are too numerous from a psychometric point of view and cannot be evaluated and reported separately by PISA. In this respect, it was stated that a Formulate—Employ—Interpret scheme assessment was created for mathematical process skills in PISA 2012. Considering these situations, since there will be difficulties in the classification performed, mathematics proficiency levels can be defined with the skill sets required by the problems (OECD, 2009). In this direction, the problems are categorized according to three competency clusters: *reproduction, connections* and *reflection* (Dossey, McCrone, & O'Sullivan, 2006; Neidorf, Binkley, Gattis, & Nohara, 2006; OECD, 2009; Altun, 2014). While this categorization was found in PISA 2003, where mathematical literacy was the focus, such a categorization was not included in the classification of problems in PISA 2012, where mathematical literacy was the focus of practice. In this regard, in the following sections, classification according to mathematical content, real life contexts and mathematical process skills is given in detail.

4.1 Classification of PISA Mathematical Literacy Problems in terms of Mathematical Content

One of the sub-dimensions of mathematical literacy measurement and evaluation in PISA applications is mathematical content knowledge (OECD, 2003; 2006; 2009; 2012; 2015; 2018; 2019b). Mathematical content includes subject areas from which the underlying content area is chosen due to the nature of the problems. This situation refers to the fields of mathematics that will apply for the solutions of the problems (Stacey & Turner, 2015). Understanding mathematical content and applying this knowledge to solving problems in personal, occupational, societal and scientific contexts has an important place for individuals in the modern world (OECD, 2019b). When it comes to mathematical content, the topics or sections that organize the field of mathematics come to mind. In the reports published after the PISA applications, classifications are made according to four subject areas or sections in terms of mathematical content in mathematical literacy. In PISA applications, each content area is 25%. These four subject areas are as follows;

1. Quantity
2. Space and Shape
3. Change and Relationship
4. Uncertainty and Data

These four categories reflect the general structure of mathematics, the main elements of the curriculum, and the mathematical events underlying mathematical problems (OECD, 2019b).

4.1.1. Quantity

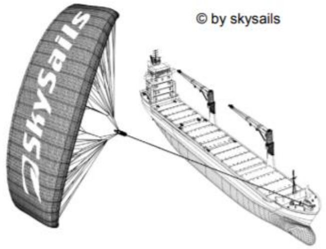
Numbers and operations unit comes to mind in this subject area. It requires understanding the sizes, perceiving the numbers, having an idea about the sizes expressed by the numbers, being aware of the number patterns, understanding the meaning of the operations, being able to calculate mathematically mentally or in writing and making predictions (MoNE, 2011; Altun, 2014). In other words, this subject area includes “sub-topic areas and actions such as numbers, number operations, mental calculations, estimation and evaluation of results” (MoNE, 2015, p. 27). It can be said that the important point in including some mathematical literacy problems in the category of quantity in terms of content area is that such problems require calculation and estimation based on numbers and operations.

The first question in the "Sailing Ships" problem, which is included in the PISA 2012 main application, can be given as an example for the classification made in the subject area of Quantity:

SAILING SHIPS

Ninety-five percent of world trade is moved by sea, by roughly 50,000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.



Question 1: SAILING SHIPS *PM923Q01*

One advantage of using a kite sail is that it flies at a height of 150 m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24 km/h is measured on the deck of the ship?

A 6 km/h
B 18 km/h
C 25 km/h
D 30 km/h
E 49 km/h

(OECD, 2013c, p. 12)

Table 1. Sailing Ships Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM923Q01	Sailing Ships Q1	2012	Simple Multiple Choice	Quantity	Scientific	Employ	%59,49	4

In the given problem, it is aimed to calculate the percentage of an event encountered in a real-life situation. This problem can be evaluated in the category of quantity according to the content area, in terms of the fact that the percentage calculation is a subject area that requires numerical operations, and the given problem requires calculations based on operations.

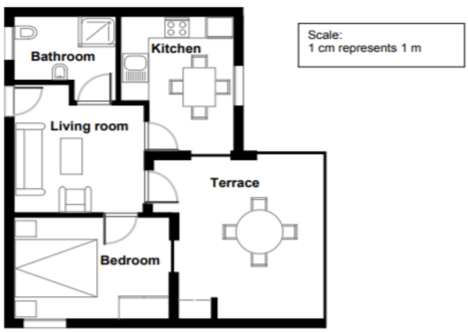
4.1.2. Space and Shape

Space and shape bring to mind the field of geometry as a content or subject area (Altun, 2014, MoNE, 2015). This subject area is encountered in a wide area in our visual world, such as the properties of objects, locations and centres, representation of objects, analysis of visual information, and coding of information (MoNE, 2011, MoNE, 2015). The field of space and shape includes activities such as map drawings, perspective drawings, drawing and rotating shapes, real life problems with shapes and objects, determining or estimating the area of a place, three-dimensional views of objects, reading and drawing maps. In terms of content, it can be said that one of the most important reasons for including mathematical literacy problems in the category of space and shape is that such problems are related to the subject area of geometry.

The problem of "Apartment Purchase", which is included in the PISA 2012 main application regarding the subject area of space and shape, can be given as an example.

APARTMENT PURCHASE

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



Translation Note: In this unit please retain metric units throughout.

Translation Note: Translate the term "real estate agency" into local terminology for businesses that sell houses.

Question 1: APARTMENT PURCHASE PM00FQ01 – 0 1 9

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

However, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the **four** lengths that are needed to estimate the total floor area of the apartment.

(OECD, 2013c, p.4)

Table 2. Apartment Purchase Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty%	ML Level
PM00FQ01	Apartment Purchase Q1	2012	Constructed Response Expert	Space and shape	Personal	Formulate	% 44,64	4

When the "Apartment Purchase" Question 1 is examined, it requires reasoning to determine the area of a place. When this problem is considered in terms of content area, it is possible to indicate that it can be evaluated within the subject area of geometry. Therefore, it can be stated that this problem is related to the category of space and shape content area.

4.1.3. Change and Relationship

Change and relations can be expressed as the mathematical reflection of the interaction and relations between objects, situations, conditions or features in an event presented in real life or in a fictional setting. In this subject area, variables and algebra come to mind (Altun, 2014). Change and relations include the creation and interpretation of symbolic or graphical representations of relations, transforming them into different forms and modelling these relations using functions (MoNE, 2011). Changes and relationships are encountered in situations such as modelling change and relationships, describing a problem situation with different representations such as functions, symbols, graphics or equations (MoNE, 2015). The subject of change and relations includes algebraic expressions, representation of tables and graphs and related functions, equations, inequalities and algebra. The important point in including some mathematical literacy problems in the category of change and relations in terms of content is that they include the representation or representation of an existing relationship or change in such problems by using a model or symbol.

The "Drip Rate" problem, which is included in the PISA 2012 main application regarding the subject area of Change and Relationships, can be given as an example.

DRIP RATE

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients.

Nurses need to calculate the drip rate, *D*, in drops per minute for infusions.

They use the formula $D = \frac{dv}{60n}$ where

d is the drop factor measured in drops per millilitre (mL)

v is the volume in mL of the infusion

n is the number of hours the infusion is required to run.

Question 1: DRIP RATE PM903Q01 – 0 1 2 9

A nurse wants to double the time an infusion runs for.

Describe precisely how *D* changes if *n* is **doubled** but *d* and *v* do not change.

....

....

....

(OECD, 2013c, p. 6-7)

Table 3. Drip Rate Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM903Q01	Drip Rate Q1	2012	Constructed Response Expert	Change and relationships	Occupational	Employ	%22,23	5

When the "Drip Rate" problem is examined, it is requested to explain the effect of doubling a data in the formula and keeping the other data constant on the resulting value. This problem seems to be related to explaining the relationship between the data and the relationship between the change in the context of the problem and other variables. It can be said that this problem, in which relations are formulated algebraically, is related to the content area of change and relations.

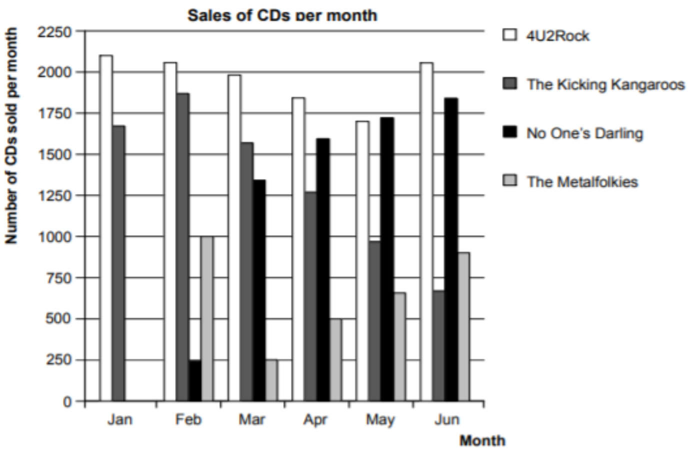
4.1.4. Uncertainty and Data

Uncertainty and data incudes probability and statistics (Altun, 2014; MoNE, 2015). The subject of uncertainty has an important place in the evaluation of mathematical analysis and probabilities in real life situations. This subject area includes processes such as collecting data, being able to analyse data, making comments based on the given data, processing data, evaluating conditions, and expressing diversity quantitatively. In terms of content, it can be said that one of the most important reasons why mathematical literacy problems are in the field of uncertainty and data is that the data or data group given in the problem requires interpretation and inferences from the data.

The "Charts" problem, which is included in the PISA 2012 main application regarding the subject area of Uncertainty and Data can be given as an example.

CHARTS

In January, the new CDs of the bands *4U2Rock* and *The Kicking Kangaroos* were released. In February, the CDs of the bands *No One's Darling* and *The Metalfolkies* followed. The following graph shows the sales of the bands' CDs from January to June.



Translation Note: The term "charts" does not refer to the mathematical term, but to the weekly listing of the best selling music CDs.

Translation Note: Translate band names with fictitious band names in your language.

Translation Note: The names of the months are shown in abbreviated form in the graphic. Full names can be used if space allows, as shown in the FRE version.

Question 1: CHARTS *PM918Q01*

How many CDs did the band *The Metalfolkies* sell in April?

A 250
B 500
C 1000
D 1270

(OECD, 2013c, p.9)

Table 4. Charts Question 1 Information

Item Code	Item Name	PISA Source	Item forma	Content category	Context category	Process category	Item Difficulty	ML Level
PM918Q01	Charts Q	2012	Simple Multiple Choice	Uncertainty and data	Societal	Interpret	%87,27	Below Level 1

It is aimed to reach the correct answer by reading the graph given in the "Charts" Question 1. This problem seems to be related to making inferences from a given data set. In this respect, it can be said that the "Charts" Question 1 is located in the field of Uncertainty and Data content.

4.2. Classification of PISA Mathematical Literacy Problems in Terms of Contexts

Another area in which mathematical literacy problems are classified is the real-world context of the problems. The context presented in the problems is expressed as the world of the individual involved in the problem (MoNE, 2011). Choosing the appropriate strategy for the problem or creating the representations for the problem is generally context dependent, and developing a model in this context requires the use of real-world context (OECD, 2019b). In terms of context, problems related to four categories, namely personal, occupational, societal and scientific, are encountered with mathematical literacy problems. In addition, when the problems in PISA applications are examined in terms of context, it is stated that the number of problems in personal, occupational, societal and scientific contexts is equally distributed (Altun, 2014). In PISA, the distribution of problems in terms of each context is 25%. In addition, while contexts contribute to the development of real-world problems, there is no attitude towards making assessments in PISA according to these contexts.

4.2.1. Personal

It is seen that the problems in the personal context are generally presented in relation to the person himself, his family and his life. It is seen that such problems are related to food preparation, shopping, games, personal health, time management, budget planning, and travel (MoNE, 2011; Altun, 2014). Problems in the personal context arise from real-life situations from the perspective of individuals in the centre (Stacey & Turner, 2015). The "Sauce" problem in the PISA 2012 main application can be given as an example of the personal context below:

SAUCE

Question 2: SAUCE *PM924Q02 – 0 1 9*

You are making your own dressing for a salad.

Here is a recipe for 100 millilitres (mL) of dressing.

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing?

Answer: ………………... mL

(OECD, 2013c, p.16).

Table 5. Sauce Question 2 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM924Q02	Sauce Q2	2012	Constructed Response Manual	Quantity	Personal	Formulate	%63,45	3

When the "Sauce" Question 2 is examined, it is seen that a person prepares a sauce individually in the context of real life. It can be said that this problem is a problem in a personal context in terms of both the individual's own life and presenting it in a context related to individual food preparation.

4.2.2. Occupational

It can be said that the problems expressed in the professional context are generally business-oriented and presented in the context of the knowledge and skills required by a profession. Such problems are generally encountered as problems involving business life such as design-architecture, business contents, time management, business analysis, accounting, and ordering for buildings (MoNE, 2011; Altun, 2014). Questions in the occupational context are formed from the business world (Stacey & Turner, 2015). “Garage” problems, which are among the main application problems of PISA 2012, can be given as an example of the occupational context:

GARAGE

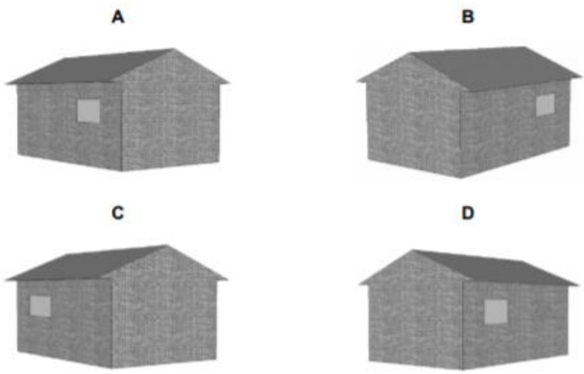
A garage manufacturer's "basic" range includes models with just one window and one door.

George chooses the following model from the "basic" range. The position of the window and the door are shown here.

Question 1: GARAGE PM991Q01

The illustrations below show different "basic" models as viewed from the back. Only one of these illustrations matches the model above chosen by George.

Which model did George choose? Circle A, B, C or D.



(OECD, 2013c, p.29)

Table 6. Garage Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM991Q01	Garage Q1	2012	Simple Multiple Choice	Space and shape	Occupational	Interpret	% 65,14	1

Since the "Garage" Question 1 is related to painting, construction or other activities of building projects, it is categorized in the Occupational context. Because the problem is created in the context of the construction, painting or other completion of a construction project. Problems in the occupational category are usually created in the context of situations such as design, architecture, and building construction. For these reasons, it can be said that the "Garage" problem is in the occupational category in terms of context.

4.2.3. Societal

Problems in the societal context consist of being citizen, local, national or universal (Stacey & Turner, 2015). Problems focus on the society in which the individual lives. The problems presented in this context include issues such as the election system, government policies, economy population movements, national statistics and economy related to societies (MoNE, 2011; Altun, 2014). The "Climbing Mount Fuji" problem, which is included in the PISA 2012 main application, can be given as an example of the problems given in the societal context below:

CLIMBING MOUNT FUJI

Mount Fuji is a famous dormant volcano in Japan.

Translation Note: Please do not change the names of locations or people in this unit: retain "Mount Fuji", "Gotemba" and "Toshi".

Question 1: CLIMBING MOUNT FUJI *PM942Q01*

Mount Fuji is only open to the public for climbing from 1 July to 27 August each year. About 200 000 people climb Mount Fuji during this time.

On average, about how many people climb Mount Fuji each day?

A 340
B 710
C 3400
D 7100
E 7400

(OECD, 2013c, p.19)

Table 7. Climbing Mount Fuji Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM942Q01	Climbing Mount Fuji Q1	2012	Simple Multiple Choice	Quantity	Societal	Formulate	% 46,93	2

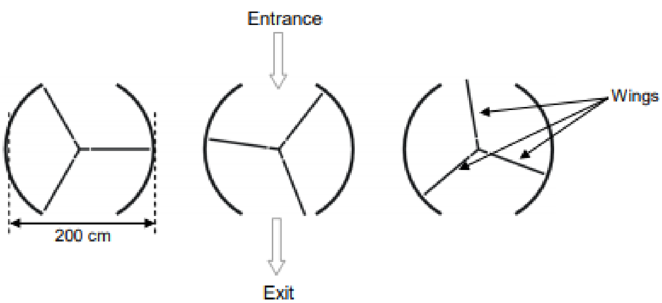
The data in the problem for "Climbing Mount Fuji" Question 1 contains information on access and public climbing to Mount Fuji and its roads. At the same time, this problem is evaluated in a societal context, as it includes a social activity towards Mount Fuji. In this respect, it can be stated that the "Climbing Mount Fuji" is a problem presented in the societal context.

4.2.4. Scientific:

It can be said that the problems in the scientific context include mathematical applications related to science and technology. Such problems are problems that require a scientific subject area such as mathematical proof and proof, weather conditions, climate, environment, genetics, space sciences. Mathematical analysis are used for science and technology in scientific problems (Stacey & Turner, 2015). With the PISA 2012 application, it can be said that the questions in the scientific context also include the problems about mathematical structures. In the scientific context, the "Revolving Door" problem, which is included in the PISA 2012 main application, can be given as an example to this category:

REVOLVING DOOR

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.



Translation Note: If the term for "wings" in the context of a revolving door is not familiar to 15-year olds in your country, you may wish to introduce the term as for example in the FRE source version: "Une porte à tambour est composée de trois « ailes », appelées vantaux, qui tournent au sein d'un espace circulaire."

Question 1: REVOLVING DOOR *PM995Q01 – 0 1 9*

What is the size in degrees of the angle formed by two door wings?

Size of the angle: ....°

(OECD, 2013c, p.33)

Table 8. Revolving Door Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM995Q01	Revolving Door Q1	2012	Constructed Response Manual	Space and shape	Scientific	Employ	% 57,67	3

It is seen that the "Revolving Door" problem, although not clearly, includes concepts related to engineering and was created in the context of scientifically given information. From this point of view, it can be said that this problem can be evaluated in a scientific context.

4.3. Classification of Mathematical Literacy Problems According to Process Skills

It is seen that with the PISA applications, the competence and skill structures have been replaced by the term "mathematical processes" (Niss, 2015). Mathematical process skills are important for both learning mathematics and understanding mathematical content, concepts and problems (Kaosa-ard, Erawan, Damrongpanit, & Suksawang, 2015). These skills allow students to solve problems accurately and logically. Process skills begin when faced with real life or problem situations in the context of other disciplines such as physics and biology (Gatabi, Stacey, & Gooya, 2012). By using these skills effectively, students master the mathematical language by making connections between real-life situations and the mathematical world. It can be said that mathematical process skills play an important role in raising students as problem solvers. Because, transferring a problem encountered in real life situations to mathematical language includes the process of formulating, solving a problem in the mathematical world includes employing, and the results are interpreted and evaluated. When students internalize these processes, they can overcome problems as problem solvers. In this respect, mathematical processes are seen as abilities in which mathematics is applied in various situations (Kaosa-ard et al., 2015) because these situations require mathematical thinking. It is stated that mathematical thinking activities are effective in the acquisition and understanding of mathematical processes (Lew, Cho, Koh, Koh, & Paek, 2012). It can be said that mathematics subjects are also related to mathematics process skills (MoNE, 2013). In this respect, it is considered important for an effective education that the activities used in students' mathematics learning will improve their mathematical process skills (Reys, Lindquist, Lambdin, Smith, & Suydam, 2007). Mathematical literacy of students can be improved by activating the basic mathematical competences that form the basis of formulating, employing and interpreting processes (Dewantara, Zulkardi, & Darmawijoyo, 2015).

With the PISA 2012 application, mathematical process skills are more evident; It has been clearly drawn into a new definition as 1- *Formulating* situations mathematically, 2- *Employing* mathematical concepts, facts, procedures and reasoning, and 3- *Interpreting*, applying and evaluating mathematical outcomes (Stacey, 2012). It can be said that this situation emerged as a result of updating the mathematical structure of PISA in the PISA 2012 application (Kelly, Nord, Jenkins, Chan, & Kastberg, 2013). Kelly et al. (2013) state that these new definitions are more meaningful and in a real context. Tout and Spithill (2015), on the other hand, indicate that it was not easy to distinguish sharply the lines between processes in previous PISA applications and state that the test developers and the mathematics expert group introduced a new classification in the PISA 2012 application. These three processes present a useful and meaningful system in the organization and classification of mathematical processes, which reveals what students do when they relate to a mathematical problem they encounter and reach the solution of the problem (MoNE, 2015). As a result, three mathematical processes are defined in the measurement and evaluation of mathematical literacy in the PISA 2012 application (MoNE, 2015):

1- *Formulating* situations mathematically,
2- *Employing* mathematical concepts, facts, procedures and reasoning,
3- *Interpreting*, applying and evaluating mathematical outcomes

4.3.1. Formulating situations mathematically

Mathematical literacy usually starts with the problem in context (Stacey & Turner, 2015). The problem solver determines the relationship between the problem situation and mathematics. In the next step after determining the relationship, it tries to formulate the situations mathematically by using mathematical concepts, trying to simplify the assumptions and define the relationships. In other words, a problem in a context is tried to be transformed into a mathematical problem. This process followed by the problem solver is expressed as the process of formulating mathematical situations. Formulating is defined as "determining the mathematical view of a problem in the real world and its meaningful variables, identifying mathematical structures, transferring the problem to mathematical language and appearance, simplifying problems and situations in order to make mathematical analysis, mathematically displaying situations using variables, symbols, shapes and models, showing a problem in different ways, establishing the relationship of the problem with known problems or mathematical concepts or processes, and illustrating mathematical relations arising from a conceptual problem through the use of technology (MoNE, 2011, p.16). The formulating process includes the following behaviours (OECD, 2013b, p.28):

- *identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables;*
- *recognising mathematical structure (including regularities, relationships, and patterns) in problems or situations;*
- *simplifying a situation or problem in order to make it amenable to mathematical analysis;*
- *identifying constraints and assumptions behind any mathematical modelling and simplifications gleaned from the context;*
- *representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models;*
- *representing a problem in a different way, including organising it according to mathematical concepts and making appropriate assumptions;*
- *understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically;*
- *translating a problem into mathematical language or a representation;*
- *recognising aspects of a problem that correspond with known problems or mathematical concepts, facts, or procedures; and*
- *using technology (such as a spreadsheet or the list facility on a graphing calculator) to portray a mathematical relationship inherent in a contextualised problem*

In general, it is seen that the formulating process includes behaviours such as the determination of the mathematical appearance of the problems and their meaningful variables, the determination of mathematical structures, the transformation of the problem into mathematical language and appearance, the mathematical representation of situations using variables, symbols, shapes and models, the relationship of the problem with mathematical concepts or processes, and illustrating mathematical relationships through the use of technology. The "Sailing Ships" Question 4 in the PISA 2012 main application can be given as an example of problems that require the process of formulating.

Question 4: SAILING SHIPS *PM923Q04 – 0 1 9*

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship *NewWave* are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: *NewWave*

Type: freighter

Length: 117 metres

Breadth: 18 metres

Load capacity: 12 000 tons

Maximum speed: 19 knots

Diesel consumption per year without a kite sail: approximately 3 500 000 litres

The cost of equipping the *NewWave* with a kite sail is 2 500 000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail? Give calculations to support your answer.

Number of years:......................................

(OECD, 2013c, p.14)

Table 9. Sailing Ships Question 4 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM923Q04	Sailing Ships Q4	2012	Constructed Response Expert	Change and relationships	Scientific	Formulate	% 15,28	6

When the "Sailing Ships" Question 4 is examined, it requires multi-step modelling by transforming the problem given in a context into a mathematical context. In the given problem, this problem can be evaluated in the formulating process, as it requires the calculation of one year's fuel consumption, how much the savings are made in a year by installing a parachute, and how much the savings made in one year cover the parachute cost, and to formulate the data mathematically.

4.3.2. Employing mathematical concepts, facts, procedures and reasoning

The process of employing mathematical concepts, facts, procedures and reasoning includes the mathematical results of abstract objects in the mathematical world (Stacey & Turner, 2015). It requires interpretation of mathematical results according to real problem situations, which include the results of mathematically formulated problems in a context. In other words, in order for the problem solver to derive mathematical results from a mathematically formulated problem situation in a context, he must apply to some situations such as applying mathematical rules, developing strategies, mathematical facts and reasoning. In order to solve mathematically formulated problems involving mathematical results, mathematical concepts, facts and reasoning must be employed (Edo, Hartono & Putri 2013). The process of employing involves problem solvers applying the mathematical operations necessary to find a mathematical solution and obtain results. While the key idea in PISA is expressed as the separate reporting of process movements between the real world and the mathematical world, running is seen as a working process within the mathematical world (Stacey & Turner, 2015).

In the PISA 2012 national final report published by the Ministry of National Education (2015: 21), the process of *employing* mathematical concepts, facts, procedures and reasoning is defined as “the situation in which individuals use mathematical, concept, fact, operation and reasoning in solving mathematically formulated problems in order to obtain some mathematical decisions”. The employing process requires skills such as solving equations, making arithmetic calculations, reading tables and graphs, making mathematical assumptions or inferences from tables and graphs, and displaying figures.

The employing process includes the following behaviours (OECD, 2013b, p. 29):

- *Devising and implementing strategies for finding mathematical solutions;*
- *using mathematical tools, including technology, to help find exact or approximate solutions;*
- *applying mathematical facts, rules, algorithms, and structures when finding solutions;*
- *manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations;*
- *making mathematical diagrams, graphs, and constructions and extracting mathematical information from them;*

- *using and switching between different representations in the process of finding solutions;*
- *making generalisations based on the results of applying mathematical procedures to find solutions; and*
- *reflecting on mathematical arguments and explaining and justifying mathematical results.*

The employing process can be expressed as the process in which mathematical methods are used to obtain mathematical results and solutions. It is seen that it requires behaviours such as making arithmetic calculations, solving inequalities, making logical inferences from mathematical assumptions, and deducing mathematical knowledge from tables and graphs. "Helen The Cyclist" Question 1 in the main application of PISA 2012 can be given as an example of problems that require the process of *employing*:

HELEN THE CYCLIST

Helen has just got a new bike. It has a speedometer which sits on the handlebar.

The speedometer can tell Helen the distance she travels and her average speed for a trip.

Question 1: HELEN THE CYCLIST PM957Q01

On one trip, Helen rode 4 km in the first 10 minutes and then 2 km in the next 5 minutes.

Which one of the following statements is correct?

A. Helen's average speed was greater in the first 10 minutes than in the next 5 minutes.
B. Helen's average speed was the same in the first 10 minutes and in the next 5 minutes.
C. Helen's average speed was less in the first 10 minutes than in the next 5 minutes.
D. It is not possible to tell anything about Helen's average speed from the information given.

(OECD,2013, p.22)

Table 10. Helen The Cyclist Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM957Q01	Helen Th Cyclist Q	2012	Simple Multiple Choice	Change an relationships	Personal	Employ	% 52,91	2

When the "Helen The Cyclist" problem is examined, it is seen that the answers are related to the average speed, and to solve the problem, the average speed in the first 10 minutes and the next 5 minutes should be calculated and compared. In calculating the average velocity, arithmetic calculation is required. The correct answer can be obtained by comparing the average speeds calculated for the first 1 minute and the next 5 minutes. The realization of these operations requires basic arithmetic calculation and comparison skills. In this respect, the "Helen The Cyclist" problem requires the process of employing.

4.3.3. Interpreting, applying and evaluating mathematical outcomes

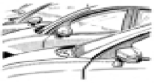
Modelling processes, which start with a problem situation encountered in the context of real life, continue with the formulation of the problem through mathematical terms and concepts and its transfer to the mathematical world (Gatabi, Stacey, & Gooya, 2012). The mathematically formulated problem is solved by applying mathematical concepts and methods. It is necessary to transfer mathematical concepts, facts, procedures and reasoning from the mathematical world to the real world in order to evaluate the problem solved in the mathematical world in the context of real life and to determine the accuracy of the result obtained (Stacey & Turner, 2015). It can be said that the process of interpreting is effective in transferring mathematical outputs from the mathematical world to real life situations. In the interpretation phase, problem solvers pay attention to the mathematical results and check the meaning of the results in real life context and whether they are sufficient against the requirements of the real problem (OECD, 2013b).

In the PISA report published by the Ministry of National Education (2011, p. 17), the process of interpreting includes “re-interpreting the mathematical result in the real world, evaluating the suitability of the mathematical solution in the context of the problem encountered in the real world, the effects of the outputs of a mathematical process or model in the real world, understanding the limits of the mathematical concepts and solutions, determining the limits of the model used to solve the problem and criticizing it”. On the other hand, OECD (2013b, p. 29) defines interpretation in the PISA 2012 evaluation report as “the abilities of individuals to reflect upon mathematical solutions, results, or conclusions and interpreting them in the context of real-life problems". This process also includes implementation and evaluation processes. In order to be able to evaluate, the results must first be obtained. The results obtained can be evaluated or practiced by transferring them to real life situations. The process of interpreting includes the following behaviours (OECD, 2013b, p.29-30):

- *interpreting a mathematical result back into the real world context;*
- *evaluating the reasonableness of a mathematical solution in the context of a real-world problem;*
- *understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgments about how the results should be adjusted or applied*
- *explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem;*
- *understanding the extent and limits of mathematical concepts and mathematical solutions; and*
- *critiquing and identifying the limits of the model used to solve a problem.*

It is seen that the interpreting process includes behaviours such as reinterpreting the mathematical result in the real world, evaluating the mathematical solution in the context of the problem encountered in the real world, and the effects of the mathematical process or the outputs of the model in the real world. The emphasis in the interpretive process is thought to be in the context of the real world. In other words, it can be said that it involves the interpretation process of problems that require problem solvers to interpret, evaluate or make inferences in the context of real-life situations. The "Which Car" Question 1 in the PISA 2012 main application can be given as an example of the process of interpreting:

WHICH CAR?



Chris has just received her car driving licence and wants to buy her first car.

This table below shows the details of four cars she finds at a local car dealer.

Model:	**Alpha**	**Bolte**	**Castel**	**Dezal**
Year	2003	2000	2001	1999
Advertised price (zeds)	4800	4450	4250	3990
Distance travelled (kilometres)	105 000	115 000	128 000	109 000
Engine capacity (litres)	1.79	1.796	1.82	1.783

Translation Note: Change the car's names to other more suitable fictional names if necessary – but keep the other numbers and values the same.

Translation Note: The use of zeds is important to the Unit, so please do not adapt "zed" into an existing currency.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Question 1: WHICH CAR? PM985Q01

Chris wants a car that meets **all** of these conditions:

- The distance travelled is **not** higher than 120 000 kilometres.
- It was made in the year 2000 or a later year.
- The advertised price is **not** higher than 4500 zeds.

Which car meets Chris's conditions?

A. Alpha
B. Bolte
C. Castel
D. Dezal

(OECD, 2013c, p.26)

Table 11. Which Car Question 1 Information

Item Code	Item Name	PISA Source	Item format	Content category	Context category	Process category	Item Difficulty	ML Level
PM985Q01	Which Car Q1	2012	Simple Multiple Choice	Uncertainty and data	Personal	Interpret	% 81,14	Below level 1

In the solution of the "Which Car" Question 1, it requires an evaluation in line with the limitations given in the context of the problem, based on the mathematical information given about the vehicles. Commenting on the classification of the problem, he must first eliminate the "Castel" tool in line with the first classification, and conclude that the "Dezal" tool cannot be selected in line with the second restriction and the "Alpha" tool in the third condition. If these limitations are evaluated as a mathematical output, the solution process of the problem requires reaching the correct answer by interpreting and evaluating these outputs. Based on the stated reasons, the "Which Car" Question 1 is classified within the process of interpreting.

5. Mathematical Literacy Proficiency Levels

Mathematical literacy levels are reported according to well-defined levels that reveal students' proficiency in line with the analysis of the answers given to mathematical literacy problems by students in countries participating in PISA (OECD, 2014). By classifying PISA mathematical literacy in 6 levels, a scale was put forward by making a detailed description of each proficiency level. In the definitions made for the proficiency level, it is stated what the students at a certain proficiency level can and cannot do. In addition, by defining the score ranges for the proficiency levels, the mathematical literacy levels of the students and, accordingly, the mathematical literacy levels and scores of the countries participating in the PISA applications are determined. The proficiency levels determined for PISA mathematical literacy and the situations that students at these levels can achieve are presented in the table below (OECD, 2013b, p.41):

Table 12. Proficiency scale Descriptions for Mathematics Literacy

Level	
6	At Level 6 students can conceptualise, generalise and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply their insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.
5	At Level 5 students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
4	At Level 4 students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.

3	At Level 3 students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications when reporting their interpretations, results and reasoning.
2	At Level 2 students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
1	At Level 1 students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

6-Conclusion

In this chapter, mathematical literacy and categorization of mathematical literacy problems are explained with examples within the framework of the aims, objectives and importance of PISA.

Mathematics is at the centre of curricula all over the world (Turner, 2016). In today's world, where individuals' attitudes towards mathematics and their level of use of mathematics are gaining more and more importance, PISA sheds light on the mathematics education of countries by evaluating mathematical literacy and the extent to which students can use the information they learned at school in real life. More than 90 countries and more than 2 million students have participated in PISA since 2000. It is obvious that this rate will continue to increase. In this context, it is important to analyse the PISA applications and interpret the results in order to evaluate the readiness of the education systems of the countries for the future, which provides a wide assessment framework in other fields, especially in reading, science literacy and mathematical literacy at the international level.

Problems encountered in real life in the 21st century are becoming more and more difficult and complex. In this context, it is important for each individual to realize, understand and know the role of mathematics in real life in order to overcome complex problems. Mathematical literacy is a basic requirement for individuals to be able to work effectively in today's world, since they have the capacity to know and understand the role mathematics plays in real life and to use and interpret mathematical knowledge and skills in real life situations (National Research Council, 1989). The aim of mathematics education in schools is to prepare individuals for real life. In this direction, among the main objectives of mathematics teaching programs are the learners to understand mathematical concepts and use these concepts in daily life, and to develop and effectively use mathematical literacy skills (MoNE, 2018). In order to improve students' mathematical literacy levels, real-life problems should be included in learning environments, mathematical literacy problems that are accessible in PISA applications should be analysed, and discussions should be held on problems in terms of content, context and mathematical processes.

In mathematics lessons in schools, there is not enough time to work with real-world situations and real-life problems and to explore different solutions for these problems (Turner, 2016). Therefore, in the face of problems presented in a context in mathematics lessons or in international exams such as PISA, it is seen that many students face a serious challenge that they do not know what to do, and that they have difficulty in using the information they learned at school and making connections between information. The fact that students encounter more mathematical literacy problems in the mathematics learning environment at school will enable them to gain experience in solving real-life problems. The mathematical literacy problem pool in the literature should not only be accessed within the framework of PISA applications, but also should be enriched by contributing to the literature with scientific studies and mathematical literacy problems that teachers will develop by considering content, context and processes. Developed problems should be used in mathematics learning environments to contribute to students' mathematical literacy level.

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Appendix 1

Table 13. Countries Participating in PISA by Year

		2000	2003	2006	2009	2012	2015	2018	2021****
	OECD countries & Associates								
1	Australia	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
2	Austria	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
3	Belgium	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4	Brazil	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
5	Canada	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
6	Chile			Yes	Yes	Yes	Yes	Yes	Yes
7	Colombia			Yes	Yes	Yes	Yes	Yes	Yes
8	Czech Republic	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
9	Denmark	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
10	Estonia			Yes	Yes	Yes	Yes	Yes	Yes
11	Finland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
12	France	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
13	Germany	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
14	Greece	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
15	Hungary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
16	Iceland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
17	Ireland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
18	Israel			Yes	Yes	Yes	Yes	Yes	Yes
19	Italy	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
20	Japan	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
21	Korea	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
22	Latvia	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
23	Lithuania			Yes	Yes	Yes	Yes	Yes	Yes
24	Luxembourg	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
25	Mexico	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
26	Netherlands	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
27	New Zealand	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
28	Norway	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
29	Poland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
30	Portugal	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
31	Slovak Republic		Yes	Yes	Yes	Yes	Yes	Yes	Yes
32	Slovenia			Yes	Yes	Yes	Yes	Yes	Yes
33	Spain	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
34	Sweden	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
35	Switzerland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
36	Thailand		Yes	Yes	Yes	Yes	Yes	Yes	Yes
37	Turkey		Yes	Yes	Yes	Yes	Yes	Yes	Yes

38	United Kingdom (except Scotland)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
39	United States	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Partners								
40	Albania				Yes	Yes	Yes	Yes	Yes
41	Algeria						Yes		
42	Argentina			Yes	Yes	Yes	Yes	Yes	Yes
43	Azerbaijan			Yes	Yes			Yes	Yes
44	Belarus							Yes	Yes
45	Bosnia & Herzegovina							Yes	Yes
46	Brunei Darussalam							Yes	Yes
47	Bulgaria			Yes	Yes	Yes	Yes	Yes	Yes
48	China (People's republic)*****				Yes	Yes	Yes	Yes	Yes
49	Costa Rica					Yes	Yes	Yes	Yes
50	Croatia			Yes	Yes	Yes	Yes	Yes	Yes
51	Dominican Republic						Yes	Yes	Yes
52	El Salvador								Yes
53	Georgia						Yes	Yes	Yes
54	Guatemala								Yes
55	Hong Kong-China		Yes	Yes	Yes	Yes	Yes	Yes	Yes
56	India**								Yes
57	Indonesia		Yes	Yes	Yes	Yes	Yes	Yes	Yes
58	Jamaica								Yes
59	Jordan			Yes	Yes	Yes	Yes	Yes	Yes
60	Kazakhstan				Yes	Yes	Yes	Yes	Yes
61	Kosovo						Yes	Yes	Yes
62	Kyrgyz Republic			Yes	Yes				
63	Lebanon						Yes	Yes	Yes
64	Macao-China		Yes	Yes	Yes	Yes	Yes	Yes	Yes
65	North Macedonia						Yes	Yes	Yes
66	Madagascar								
67	Malaysia					Yes	Yes	Yes	Yes
68	Malta						Yes	Yes	Yes
69	Mauritius								
70	Miranda								
71	Moldova						Yes	Yes	Yes
72	Mongolia								Yes
73	Montenegro*		Yes	Yes	Yes	Yes	Yes	Yes	Yes
74	Morocco							Yes	Yes
75	Panama				Yes			Yes	Yes
76	Paraguay								Yes
77	Peru				Yes	Yes	Yes	Yes	Yes
78	Philippines							Yes	Yes

79	Qatar			Yes	Yes	Yes	Yes	Yes	Yes
80	Romania			Yes	Yes	Yes	Yes	Yes	Yes
81	Russian Federation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
82	Republic of Serbia*		Yes	Yes	Yes	Yes		Yes	Yes
83	Saudi Arabia							Yes	Yes
84	Scotland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
85	Singapore				Yes	Yes	Yes	Yes	Yes
86	Chinese Taipei			Yes	Yes	Yes	Yes	Yes	Yes
87	Trinidad & Tobago				Yes		Yes		
88	Tunisia		Yes	Yes	Yes	Yes	Yes		
89	UAE***				Yes	Yes	Yes	Yes	Yes
90	Ukraine							Yes	Yes
91	Uruguay		Yes	Yes	Yes	Yes	Yes	Yes	Yes
92	Uzbekistan								Yes
93	Viet Nam					Yes	Yes	Yes	Yes
	Total	**32**	**41**	**57**	**65**	**64**	**72**	**79**	**85**

* In PISA 2003 participated as one 'Serbia and Montenegro'
**Tamil Nadu & Himachal Pradesh in 2009+, Chandigarh in 2021.
*** Dubai participated in 2009. The rest of the country in PISA 2009+. The entire country participated in the following cycles.
****As of 04 May 2020
***** Shanghai has participated in PISA since 2009. China expanded its participation with Beijing, Jiangsu and Guangdong in PISA 2015 and with Beijing, Jiangsu and Zhejiang in PISA 2018 and PISA 2021 in addition to Shanghai.
Note: Table 13. Countries Participating in PISA by Year retrieved from https://www.oecd.org/pisa/aboutpisa/pisa-participants.htm

CHAPTER II

VOLKOV TYPE AND WEIGHTED APPROXIMATION THEOREMS FOR BIVARIATE *q*-STANCU-BETA TYPE OPERATORS

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1. Introduction

Stancu (1995) introduced the following operators with the help of Lebesgue integrable functions on (0,∞) by

$$L_n(f,x)=\frac{1}{B(nx,n+1)}\int_0^\infty \frac{t^{nx-1}}{(1+t)^{nx+n+1}}\,dt.$$

Later, Abel and Gupta (2004) estimated the rate of convergence of these operators for functions of bounded variations. Afterwards, Gupta et al. (2005) obtained the rate of convergence of these operators for functions with derivatives of bounded variation, respectively. q-Analogue of the Stancu-Beta operators was also defined by Aral and Gupta (2002).

Firstly, we recall the certain notations of q-calculus. q-Integers $[k]_q$ is defined for non-negative integer *k* by

$$[k]_q:=\begin{cases}\dfrac{1-q^k}{1-q} & ,q\neq 1\\ k & ,q=1.\end{cases}$$

q-Factorial of q-integers $[k]_q$ is defined as

$$[k]_q!:=\begin{cases}[k]_q[k-1]_q\ldots\ldots[1]_q & ,k=1,2,\ldots,\\ 1 & ,k=0.\end{cases}$$

Let n, k such that $0 \leq k \leq n$ be non-negative integers. q-Binomial coefficients are defined as

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{[n]_q!}{[k]_q!\,[n-k]_q!}.$$

For $A > 0$, q-improper integral depending on A is defined by

$$\int_0^{\infty/A} f(x) d_q x = (1-q) \sum_{n=-\infty}^{\infty} f\left(\frac{q^n}{A}\right) \frac{q^n}{A}.$$

q-Beta functions including q-improper integral is introduced as follows:

$$B_q(t,s) = K(A,t) \int_0^{\infty/A} \frac{x^{t-1}}{(1+x)_q^{t+s}} d_q x.$$

Here, the notation of q-Pochhammer symbol is denoted by

$$(a+b)_q^n = \prod_{j=0}^{n-1} \left(a + q^j b\right).$$

Furthermore,

$$K(x,t) = \frac{1}{x+1} x^t \left(1 + \frac{1}{x}\right)_q^t (1+x)_q^{1-t},$$

and the following recurrence relation holds:

$$K(A, t+1) = q^t K(A,t).$$

For $t, s > 0$, another definition of q-Beta function is defined by

$$B_q(t,s) = \int_0^1 x^{t-1} (1-qx)_q^{s-1} d_q x.$$

The relation between q-beta function and q-gamma function is known as with the following equality:

$$B_q(t,s)=\frac{\Gamma_q(t)\Gamma_q(s)}{\Gamma_q(t+s)}.$$

The extensive details about q-calculus can be found in the reference (Kac&Cheung, 2002).

For $x\in[0,\infty)$, $q\in(0,1)$, q-Stancu-Beta operators are defined by Aral and Gupta (2002) as follows:

$$L_n^q(f,x)=\frac{K(A,[n]_q x)}{B_q([n]_q x,[n]_q+1)}\int_0^{\infty/A}\frac{u^{[n]_q x-1}}{(1+u)_q^{[n]_q x+[n]_q+1}}f\left(q^{[n]_q x}u\right)d_q u. \tag{1}$$

Aral and Gupta (2002) calculated the following equalities:

$$L_n^q(1,x)=1, \tag{2}$$

$$L_n^q(t,x)=x, \tag{3}$$

$$L_n^q(t^2,x)=\frac{([n]_q x+1)x}{q([n]_q-1)}, \tag{4}$$

and they also obtained m-th order moment as follows:

$$L_n^q(t^m,x)=\frac{\Gamma_q([n]_q x+m)\Gamma_q([n]_q-m+1)}{\Gamma_q([n]_q x)\Gamma_q([n]_q+1)q^{m(m-1)/2}}. \tag{5}$$

In this chapter, we build up Volkov type and weighted approximation theorems for the bivariate q-Stancu-Beta type operators.

2. Definition of Bivariate Operators

In this part, we consider bivariate q-Stancu-Beta type operators and give some auxiliary results.

Definition 1. Let be $\mathbb{R}_+ := [0,\infty)$, $A_1, A_2 > 0$, $q_1, q_2 \in (0,1)$ and h be a bivariate continuous and bounded function defined on $\mathbb{R}_+\times\mathbb{R}_+$. For each $(x,y)\in\mathbb{R}_+\times\mathbb{R}_+$ and $n_1, n_2\in\mathbb{N}$, we define bivariate q-Stancu-Beta type operators by

$$L_{n_1,n_2}^{q_1,q_2}(h(t,s);x,y)$$

$$= \frac{K(A_1,[n_1]_{q_1}x)}{B_{q_1}([n_1]_{q_1}x,[n_1]_{q_1}+1)} \frac{K(A_2,[n_2]_{q_2}y)}{B_{q_2}([n_2]_{q_2}y,[n_2]_{q_2}+1)}$$
$$\times \int_0^{\infty/A_1}\int_0^{\infty/A_2} \frac{u^{[n_1]_{q_1}x-1}}{(1+u)_{q_1}^{[n_1]_{q_1}x+[n_1]_{q_1}+1}} \frac{v^{[n_2]_{q_2}y-1}}{(1+u)_{q_2}^{[n_2]_{q_2}y+[n_2]_{q_2}+1}}$$
$$\times h\left({q_1}^{[n_1]_{q_1}x}u,{q_2}^{[n_2]_{q_2}y}v\right)d_{q_1}u d_{q_2}v.$$

It is obvious that the bivariate q-Stancu-Beta type operators are tensor-product kind linear positive operators.

Lemma 1. The following equalities hold for q-Stancu-Beta type operators:

i. $\mathrm{L}_{n_1,n_2}^{q_1,q_2}(h(t,s);x,y) = L_{n_1}^{x}\left(L_{n_2}^{y}(h(t,s);q_2);q_1\right)$,
ii. $\mathrm{L}_{n_1,n_2}^{q_1,q_2}(h(t,s);x,y) = L_{n_2}^{y}\left(L_{n_1}^{x}(h(t,s);q_1);q_2\right)$,

where

$$L_{n_1}^{x}(h(t,s);q_1) = \frac{K(A_1,[n_1]_{q_1}x)}{B_{q_1}([n_1]_{q_1}x,[n_1]_{q_1}+1)}$$
$$\times \int_0^{\infty/A_1} \frac{u^{[n_1]_{q_1}x-1}}{(1+u)_{q_1}^{[n_1]_{q_1}x+[n_1]_{q_1}+1}} h\left({q_1}^{[n_1]_{q_1}x}u,s\right)d_{q_1}u$$

and

$$L_{n_2}^{y}(h(t,s);q_2) = \frac{K(A_2,[n_2]_{q_2}y)}{B_{q_2}([n_2]_{q_2}y,[n_2]_{q_2}+1)}$$
$$\times \int_0^{\infty/A_2} \frac{v^{[n_2]_{q_2}y-1}}{(1+v)_{q_2}^{[n_2]_{q_2}y+[n_2]_{q_2}+1}} h\left(t,{q_2}^{[n_2]_{q_2}y}v\right)d_{q_2}v\,.$$

Lemma 2. We have the following equalities for bivariate q-Stancu-Beta type operators:

$$i)\ \mathrm{L}_{n_1,n_2}^{q_1,q_2}(1;x,y) = 1,$$
$$\text{ii})\ \mathrm{L}_{n_1,n_2}^{q_1,q_2}(t;x,y) = x,$$
$$\text{iii})\ \mathrm{L}_{n_1,n_2}^{q_1,q_2}(s;x,y) = y,$$
$$\text{iv})\ \mathrm{L}_{n_1,n_2}^{q_1,q_2}(t^2;x,y) = \frac{([n_1]_{q_1}x+1)x}{q_1([n_1]_{q_1}-1)},$$

$$v)\ \mathrm{L}_{n_1,n_2}^{q_1,q_2}(s^2;x,y)=\frac{\left([n_2]_{q_2}y+1\right)y}{q_2\left([n_2]_{q_2}-1\right)}.$$

Proof. By using the Lemma 1, with tiny calculation, (i-iii) can be obtained. Along with Lemma 1, by using the formula for the *m*-th order moment given in Remark 2 of (Aral&Gupta, 2002), (iv-v) can be simply proved; therefore, we omit the proofs.

3. Volkov Type Theorem

By $C(A)$ and $B(A)$ denote us the space of all real-valued continuous and bounded functions on $A\subset\mathbb{R}^2$, respectively. $C(A)$ and $B(A)$ are Banach spaces endowed with the norm:

$$\|f\|=sup\{|f(x,y)|:(x,y)\in A\}.$$

We recall Volkov's theorem certifying uniform convergence of any sequence of bivariate linear positive operators.

Theorem 1. (Volkov, 1957) Let D, be closed and bounded region in $\mathbb{R}^2$, $\left\{L_{n_1,n_2}\right\}$ be any sequence of linear positive operators defined on $C(D)$ and $f_0=e_{00}(t,s)=1,\quad f_1=e_{10}(t,s)=t,\quad f_2=e_{01}(t,s)=s,\quad f_3=e_{20}(t,s)+e_{02}(t,s)=t^2+s^2$ be the test functions of two variable. If $\left\{L_{n_1,n_2}(f_i\ )\right\}$ converge uniformly to f_i on D for $i=0,1,2,3$ then the sequence of linear positive operators $\left\{L_{n_1,n_2}(f\ )\right\}$ converges uniformly to f on D for any $f\in C(D)$.

Let be $r_1,r_2>0$, $I_i\coloneqq[0,r_i]$ for $i=1,2$ and I: = $I_1\times I_{2,}$ $\left\{q_{1,n_1}\right\}$ and $\left\{q_{2,n_2}\right\}$ be any sequences such that $0<q_{1,n_1},q_{2,n_2}<1$ satisfying the following condition

$$\lim_{n_1\to\infty} q_{1,n_1}=\lim_{n_2\to\infty} q_{2,n_2}=1. \tag{6}$$

Now, we give Volkov type theorem for the bivariate *q*-Stancu-Beta type operators.

Theorem 2. If $\left\{q_{1,n_1}\right\}$ and $\left\{q_{2,n_2}\right\}$ are any sequences such that $q_{1,n_1},q_{2,n_2}\in(0,1)$ satisfying the condition (6) and *h* is a bivariate continuous function defined on *I*, then the sequence of bivariate *q*-Stancu-Beta type linear positive operators $\left\{L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(h)\right\}$ converges uniformly to *h* on I.

Proof. By considering Lemma 1 and (i-iii) of Lemma 2, we can easily calculate that

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}\left(f_i\ (x,y)\right)-f_i(x,y)\right|=0, i=0,1,2.$$

On the other hand, we can write

$$\left|L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}\left(f_3\ (x,y)\right)-f_3(x,y)\right|$$

$$\leq\frac{1}{q_{1,n_1}\left(1-\frac{1}{[n_1]_{q_{1,n_1}}}\right)}\left(\left(1-q_{1,n_1}+\frac{q_{1,n_1}}{[n_1]_{q_{1,n_1}}}\right)(r_1)^2+\frac{r_1}{[n_1]_{q_{1,n_1}}}\right)$$
$$+\frac{1}{q_{2,n_2}\left(1-\frac{1}{[n_2]_{q_{2,n_2}}}\right)}\left(\left(1-q_{2,n_2}+\frac{q_{2,n_2}}{[n_2]_{q_{2,n_2}}}\right)(r_2)^2+\frac{r_2}{[n_2]_{q_{2,n_2}}}\right).\tag{7}$$

By considering (7), under the condition of the hypotheses of the theorem, we obtain

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}\left(f_3\ (x,y)\right)-f_3(x,y)\right|=0.$$

Thus, all the hypotheses of Theorem 1 are verified, which completes the proof.

4. Weighted Approximation

Let $\rho(x,y)=1+x^2+y^2$ be a weight function on $\mathbb{R}^2$ and $\mathbb{R}_+ := [0,\infty)$. By B_ρ denote us the space of all functions h defined on $\mathbb{R}_+\times\mathbb{R}_+$ satisfying the following inequality:

$$|h(x,y)|\leq M_h\rho(x,y), M_h>0.$$

Let C_ρ denote the subspace of all continuous functions h of B_ρ endowed with the norm:

$$\|h\|_\rho=\sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+}\frac{|h(x,y)|}{\rho(x,y)},$$

Let C_ρ^0 denote the subspace of all functions in C_ρ such that

$$\lim_{\sqrt{x^2+y^2}\to\infty}\frac{h(x,y)}{\rho(x,y)}$$

exists finitely.

We recall the following Gadžiev's results about weighted approximation theorems.

Lemma 3. (Gadžiev, 1980) Let $\{L_{n_1,n_2}\}$ be any sequence of linear positive operators from C_ρ to B_ρ. Then there exists a real number $M > 0$ such that

$$\left\|L_{n_1,n_2}(\rho)\right\|_\rho \leq M.$$

Theorem 3. (Gadžiev, 1980) Let $f_0 = e_{00}(t,s) = 1$, $f_1 = e_{10}(t,s) = t$, $f_2 = e_{01}(t,s) = s$, $f_3 = e_{20}(t,s) + e_{02}(t,s) = t^2 + s^2$ be the test functions of two variable and $\{L_{n_1,n_2}\}$ be any sequence of linear positive operators from C_ρ to B_ρ satisfying the following conditions:

$$\lim_{n_1,n_2\to\infty} \left\|L_{n_1,n_2}(f_i) - f_i\right\|_\rho = 0, i = 0,1,2,3, \tag{8}$$

then for any function $h \in C_\rho^0$,

$$\lim_{n_1,n_2\to\infty} \left\|L_{n_1,n_2}(h) - h\right\|_\rho = 0,$$

and there exists a function $h^* \in C_\rho / C_\rho^0$, such that

$$\lim_{n_1,n_2\to\infty} \left\|L_{n_1,n_2}(h^*) - h^*\right\|_\rho \geq 1.$$

Now, we consider the following linear positive operators:

$$\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(h;x,y) = \begin{cases} L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(h;x,y), & (x,y) \in I(\alpha_{n_1},\beta_{n_2}) \\ h(x,y), & otherwise, \end{cases} \tag{9}$$

where $I(\alpha_{n_1},\beta_{n_2}) = \{(x,y): 0 \leq x \leq \alpha_{n_1}, 0 \leq y \leq \beta_{n_2}\}$, $\{\alpha_{n_1}\}$ and $\{\beta_{n_2}\}$ are sequences of real numbers such that

$$\lim_{n_1\to\infty} \alpha_{n_1} = \infty = \lim_{n_2\to\infty} \beta_{n_2}.$$

Now, we present the following weighted approximation result for the bivariate operators $\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$.

Theorem 4. Let $\{q_{1,n_1}\}$ and $\{q_{2,n_2}\}$ be any sequences such that $0 < q_{1,n_1}, q_{2,n_2} < 1$ satisfying the condition given in (6) and $\left\{\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}\right\}$ be a sequence of linear positive operators defined by (9). Then for each $h \in C_\rho^0$, we have

$$\lim_{n_1,n_2\to\infty} \left\| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(h) - h \right\|_\rho = 0.$$

Proof. We demonstrate that $\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$ maps C_ρ to B_ρ. By using Lemma 2, we can write

$$\begin{aligned}
\left\| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(\rho) \right\|_\rho &= \sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+} \frac{\left| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(1+t^2+s^2;x,y) \right|}{1+x^2+y^2} \\
&\leq 1 + \sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})} \frac{\left| 1 + L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(t^2;x,y) + L_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(s^2;x,y) \right|}{1+x^2+y^2} \\
&\leq 1 + \sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})} \frac{1}{1+x^2+y^2} \left[1 + \frac{\left([n_1]_{q_{1,n_1}} x + 1\right) x}{q_{1,n_1}\left([n_1]_{q_{1,n_1}} - 1\right)} \right. \\
&\qquad \left. + \frac{\left([n_2]_{q_{2,n_2}} y + 1\right) y}{q_{2,n_2}\left([n_2]_{q_{2,n_2}} - 1\right)} \right] \\
&\leq 1 + \left[1 + \frac{2[n_1]_{q_{1,n_1}}}{q_{1,n_1}\left([n_1]_{q_{1,n_1}} - 1\right)} + \frac{2[n_2]_{q_{2,n_2}}}{q_{2,n_2}\left([n_2]_{q_{2,n_2}} - 1\right)} \right] \\
&\leq 6.
\end{aligned}$$

Therefore $\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$ maps C_ρ to B_ρ. To complete the remain of the proof, it suffices to show that the conditions (8) of Theorem 3.

By considering Lemma 2, it is clear that

$$\lim_{n_1,n_2\to\infty} \left\| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_i) - f_i \right\|_\rho = 0, i = 0,1,2.$$

Again, by considering Lemma 2, we obtain

$$\begin{aligned}
&\left\| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3) - f_3 \right\|_\rho \\
&\quad = \sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+} \frac{\left| \tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(t^2+s^2;x,y) - (x^2+y^2) \right|}{1+x^2+y^2} \\
&\quad = \sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})} \frac{1}{1+x^2+y^2} \left\{ \left| \frac{\left([n_1]_{q_{1,n_1}} x + 1\right) x}{q_{1,n_1}\left([n_1]_{q_{1,n_1}} - 1\right)} - x^2 \right| \right.
\end{aligned}$$

$$+\left|\frac{\left([n_2]_{q_{2,n_2}}y+1\right)y}{q_{2,n_2}\left([n_2]_{q_{2,n_2}}-1\right)}-y^2\right|\Bigg\}$$

$$\leq\left\{\frac{1}{q_{1,n_1}\left(1-\frac{1}{[n_1]_{q_{1,n_1}}}\right)}\left(1-q_{1,n_1}+\frac{1+q_{1,n_1}}{[n_1]_{q_{1,n_1}}}\right)\right.$$

$$\left.+\frac{1}{q_{2,n_2}\left(1-\frac{1}{[n_2]_{q_{2,n_2}}}\right)}\left(1-q_{2,n_2}+\frac{1+q_{2,n_2}}{[n_2]_{q_{2,n_2}}}\right)\right\}.$$

The right side of the last inequality converges to zero by (6). Thus

$$\lim_{n_1,n_2\to\infty}\left\|\tilde{L}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3)-f_3\right\|_\rho=0.$$

6. Conclusion

Let $\{q_{1,n_1}\}$ and $\{q_{2,n_2}\}$ be two sequences satisfying the condition (6). Under the condition (6), we obtained the uniform convergence theorem and the convergence theorem regard to the weighted norm.

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CHAPTER III

VOLKOV TYPE AND WEIGHTED APPROXIMATION THEOREMS FOR BIVARIATE q-BASKAKOV-DURRMEYER TYPE OPERATORS

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1. Introduction

Agrawal and Thamer (1998) defined the following operators with the help of Lebesgue integrable functions on $[0, \infty)$:

$$M_n(f;x) = (n-1)\sum_{k=1}^{\infty} p_{n,k}(x)\int_0^{\infty} p_{n,k-1}(t)f(t)dt + (1+x)^{-n}f(0).$$

Here

$$p_{n,k}(x) = \binom{n+k-1}{k} x^k (1+x)^{-(n+k)}.$$

The rate of convergence of these operators was studied by Gupta (2003).

Aral and Gupta (2009) defined q-Baskakov operators. Subsequently, Gupta (2008) defined q-Durrmeyer operators. Afterwards, Aral and Gupta (2011) defined a new operator known as q-Baskakov-Durrmeyer operators. They investigated the approximation properties of these operators.

We firstly give the certain notations of q-calculus. For non-negative integer k, q-integers $[k]_q$ is defined as follows:

$$[k]_q := \begin{cases} \dfrac{1-q^k}{1-q} & ,q \neq 1 \\ k & ,q = 1. \end{cases}$$

q-Factorial of the non-negative integer k is defined as

$$[k]_q! := \begin{cases} [k]_q[k-1]_q \dots \dots .. [1]_q & , k = 1,2, \dots, \\ 1 & , k = 0. \end{cases}$$

Let n, k such that $0 \leq k \leq n$ be non-negative integers. q-binomial coefficients are defined as follows:

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{[n]_q!}{[k]_q! \, [n-k]_q!}.$$

For $A > 0$, q-improper integral depending on A is defined as follows:

$$\int_0^{\infty/A} f(x) d_q x = (1-q) \sum_{n=-\infty}^{\infty} f\left(\frac{q^n}{A}\right) \frac{q^n}{A}.$$

q-Beta functions including q-improper integral is defined as:

$$B_q(t,s) = K(A,t) \int_0^{\infty/A} \frac{x^{t-1}}{(1+x)_q^{t+s}} d_q x.$$

Here is the notation of q-Pochammer symbol is denoted by

$$(a+b)_q^n = \prod_{j=0}^{n-1} (a + q^j b).$$

Moreover,

$$K(x,t) = \frac{1}{x+1} x^t \left(1 + \frac{1}{x}\right)_q^t (1+x)_q^{1-t},$$

and the following recurrence relation holds:

$$K(A, t+1) = q^t K(A,t).$$

For $t, s > 0$, another definition of q-Beta function is defined as follows:

$$B_q(t,s) = \int_0^1 x^{t-1} (1-qx)_q^{s-1} d_q x.$$

The relation between q-beta function and q-gamma function is known as with the following equality:

$$B_q(t,s)=\frac{\Gamma_q(t)\Gamma_q(s)}{\Gamma_q(t+s)}.$$

The comprehensive details about q-calculus can be found in the reference (Kac&Cheung, 2002).

q-Baskakov-Durrmeyer type operators are defined by Agrawal and Kumar (2014) as follows:

$$M_n^q(f,x)=[n-1]_q\sum_{k=1}^{\infty}p_{n,k}^q(x)\int_0^{\infty/A}q^{k-1}p_{n,k-1}^q(t)f(t)d_qt+p_{n.0}^q(x)f(0).$$

Here $x\in[0,\infty),\ q\in(0,1)$ and

$$p_{n,k}^q(x)=\binom{n+k-1}{k}_q q^{k(k-1)/2}\frac{x^k}{(1+x)_q^{n+k}}.$$

Agrawal and Kumar (2014) calculated the following inequalities:

$$M_n^q(1;x)=1\,,$$
$$M_n^q(t;x)=\frac{[n]_qx}{q[n-2]_q},$$
$$M_n^q(t^2;x)=\frac{[n]_q[n+1]_qx^2}{q^4[n-2]_q[n-3]_q}+\frac{[2]_q[n]_qx}{q^3[n-2]_q[n-3]_q}.$$

They also proved a basic convergence theorem obtaining a recurrence relation for these operators. Furthermore, they estimated the rate of convergence these operators and obtained weighted approximation results.

In this chapter, we construct Volkov type and weighted approximation theorems for bivariate q-Baskakov-Durrmeyer type operators.

2. Definition of Bivariate Operators

In this part, we consider bivariate q-Baskakov-Durrmeyer type operators and give some auxiliary results.

Definition 1. Let be $\mathbb{R}_+:=[0,\infty)$, $A_1,A_2>0$, $q_1,q_2\in(0,1)$ and f be a bivariate continuous and bounded function defined on $\mathbb{R}_+\times\mathbb{R}_+$. For each

$(x,y) \in \mathbb{R}_+ \times \mathbb{R}_+$ and $n_1, n_2 \in \mathbb{N}$, we define bivariate q-Baskakov-Durrmeyer type operators as follows:

$$M_{n_1,n_2}^{q_1,q_2}(f(t,s);x,y) = [n_1-1]_{q_1}[n_2-1]_{q_2}\sum_{k=1}^{\infty}\sum_{j=1}^{\infty} P_{n_1,k}^{q_1}(x)\, P_{n_2,j}^{q_2}(y)$$

$$\times \int_0^{\infty/A_1}\int_0^{\infty/A_2} q_1^{k-1}q_2^{j-1}\, P_{n_1,k-1}^{q_1}(t)\, P_{n_2,j-1}^{q_2}(s) f(t,s)\, d_{q_2}s\, d_{q_1}t$$

$$+P_{n_1,0}^{q_1}(x)\, P_{n_2,0}^{q_2}(y) f(0,0).$$

Here

$$P_{n_1}^{q_1}(x) := \binom{n_1+k-1}{k}\, q_1^{k(k-1)/2}\, \frac{x^k}{(1+x)_{q_1}^{n_1+k}}, x \in \mathbb{R}_+,$$

$$P_{n_2}^{q_2}(y) := \binom{n_2+j-1}{j}\, q_2^{j(j-1)/2}\, \frac{y^j}{(1+y)_{q_2}^{n_2+j}}, y \in \mathbb{R}_+.$$

Lemma 1. The following equalities hold:

(i) $M_{n_1,n_2}^{q_1,q_2}(f(t,s);x,y) =$

$$M_{n_1}^{x}\left(M_{n_2}^{y}(f(t,s);q_2);q_1\right) + P_{n_1,0}^{q_1}(x)P_{n_2,0}^{q_2}(y)f(0,0),$$

(ii) $M_{n_1,n_2}^{q_1,q_2}(f(t,s);x,y) =$
$M_{n_2}^{y}\left(M_{n_1}^{x}(f(t,s);q_1);q_2\right) + P_{n_1,0}^{q_1}(x)P_{n_2,0}^{q_2}(y)f(0,0).$

Here

$M_{n_1}^{x}(f(t,s);q_1) :=$
$[n_1-1]_{q_1}\sum_{k=1}^{\infty} P_{n_1,k}^{q_1}(x)\int_0^{\infty/A_1} q_1^{k-1} P_{n_1,k-1}^{q_1}(t)\, f(t,y) d_{q_1}t,$

$M_{n_2}^{y}(f(t,s);q_2) :=$
$[n_2-1]_{q_2}\ \sum_{j=1}^{\infty} P_{n_2,j}^{q_2}(y)\int_0^{\infty/A_2} q_2{}^{j-1}\, P_{n_2,j-1}^{q_2}(s) f(x,s) d_{q_2}s.$

Lemma 2. We have the following equalities for bivariate q-Baskakov-Durrmeyer type operators:

i. $M_{n_1,n_2}^{q_1,q_2}(1;x,y) = 1,$

ii. $M_{n_1,n_2}^{q_1,q_2}(t;x,y) = \frac{[n_1]_{q_1}x}{q_1[n_1-2]_{q_1}},$

iii. $M_{n_1,n_2}^{q_1,q_2}(s;x,y) = \frac{[n_2]_{q_2}y}{q_2[n_2-2]_{q_2}},$

iv. $M_{n_1,n_2}^{q_1,q_2}(t^2;x,y) = \frac{[n_1]_{q_1}[n_1+1]_{q_1}x^2}{{q_1}^4[n_1-2]_{q_1}[n_1-3]_{q_1}} + \frac{[2]_{q_1}[n_1]_{q_1}x}{{q_1}^3[n_1-2]_{q_1}[n_1-3]_{q_1}},$

v. $M_{n_1,n_2}^{q_1,q_2}(s^2;x,y) = \frac{[n_2]_{q_2}[n_2+1]_{q_2}y^2}{{q_2}^4[n_2-2]_{q_2}[n_2-3]_{q_2}} + \frac{[2]_{q_2}[n_2]_{q_2}y}{{q_2}^3[n_2-2]_{q_2}[n_2-3]_{q_2}}.$

Proof. By using the Lemma 1, with small calculation, (i-v) can be obtained, therefore we omit the proof.

3. Volkov Type Theorem

By $C(A)$ and $B(A)$ denote us the space of all real-valued continuous and bounded functions on $A \subset \mathbb{R}^2$, respectively. $C(A)$ and $B(A)$ are Banach spaces endowed with the norm:

$$\|f\| = sup\{|f(x,y)|: (x,y) \in A\}.$$

We recall Volkov's theorem certifying uniform convergence of any sequence of bivariate linear positive operators.

Theorem 1. (Volkov, 1957) Let D, be closed and bounded region in $\mathbb{R}^2$, $\{L_{n_1,n_2}\}$ be any sequence of linear positive operators defined on $C(D)$ and $f_0 = e_{00}(t,s) = 1,\ f_1 = e_{10}(t,s) = t,\ f_2 = e_{01}(t,s) = s,\ f_3 = e_{20}(t,s) + e_{02}(t,s) = t^2 + s^2$ be the test functions of two variable. If $\{L_{n_1,n_2}(f_i\,)\}$ converge uniformly to f_i on D for $i = 0,1,2,3$ then the sequence of positive linear operators $\{L_{n_1,n_2}(f\,)\}$ converges uniformly to f on D for any $f \in C(D)$.

Now, we give a Volkov type theorem for the bivariate q-Baskakov-Durrmeyer type operators.

Let be $r_1, r_2 > 0$, $I_i \coloneqq [0, r_i]$ for $i = 1,2$ and I: = $I_1 \times I_2$.

Theorem 2. Let $\{q_{1,n_1}\}$ and $\{q_{2,n_2}\}$ be any sequences such that $q_{1,n_1}, q_{2,n_2} \in (0,1)$ satisfying the following condition:

$$\lim_{n_1\to\infty} q_{1,n_1} = \lim_{n_2\to\infty} q_{2,n_2} = 1. \tag{1}$$

If any $f \in C(I)$ then the sequence of bivariate q-Baskakov-Durrmeyer type linear positive operators $\left\{M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f)\right\}$ converge uniformly to f on I.

Proof. By considering Lemma 1 and (i) of Lemma 2, we can easily calculate that

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_0\ (t,s);x,y)-f_0(x,y)\right|=0.$$

By considering Lemma 1 and (ii) of Lemma 2, we have

$$\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_1\ (t,s);x,y)-f_1(x,y)\right|\le\left|\frac{[n_1]_{q_{1,n_1}}}{q_{1,n_1}[n_1-2]_{q_{1,n_1}}}-1\right|r_1$$

By (1), we obtain

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_1\ (t,s);x,y)-f_1(x,y)\right|=0.$$

Similarly, by Lemma 1 and (iii) of Lemma 2, we can calculate that

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_2\ (t,s);x,y)-f_2(x,y)\right|=0.$$

By considering Lemma 1 and (iv) and (v) of Lemma 2, we have

$$\begin{aligned}&\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3\ (t,s);x,y)-f_3(x,y)\right|\\&\le\left|\frac{[n_1]_{q_{1,n_1}}[n_1+1]_{q_{1,n_1}}}{q_{1,n_1}^4[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}-1\right|{r_1}^2\\&+\frac{[2]_{q_{1,n_1}}[n_1]_{q_{1,n_1}}}{q_{1,n_1}^3[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}r_1\\&+\left|\frac{[n_2]_{q_{2,n_2}}[n_2+1]_{q_{2,n_2}}}{q_{2,n_2}^4[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}-1\right|{r_2}^2\\&+\frac{[2]_{q_{2,n_2}}[n_2]_{q_{2,n_2}}}{q_{2,n_2}^3[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}r_2.\end{aligned}$$

Under the condition of the hypotheses of theorem, we obtain

$$\lim_{n_1,n_2\to\infty}\max_{(x,y)\in I}\left|M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3\ (t,s);x,y)-f_3(x,y)\right|=0.$$

Therefore, all the hypotheses of Theorem 1 are verified, which completes proof.

4. Weighted Approximation

Let $\rho(x,y)=1+x^2+y^2$ be a weight function on $\mathbb{R}^2$ and $\mathbb{R}_+ := [0,\infty)$. By B_ρ denote us the space of all functions f defined on $\mathbb{R}_+\times\mathbb{R}_+$ satisfying the following inequality:

$$|f(x,y)| \leq M_f\rho(x,y), M_f > 0.$$

Let C_ρ denote the subspace of all the continuous functions f of B_ρ endowed with the norm:

$$\|f\|_\rho = \sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+} \frac{|f(x,y)|}{\rho(x,y)}.$$

Let C_ρ^0 denote the subspace of all functions in C_ρ such that

$$\lim_{\sqrt{x^2+y^2}\to\infty} \frac{f(x,y)}{\rho(x,y)}$$

exists finitely.

We recall the following Gadžiev's results about weighted approximation.

Lemma 3. (Gadžiev, 1980) Let $\{L_{n_1,n_2}\}$ be any sequence of linear positive operators from C_ρ to B_ρ. Then there exists a real number $M>0$ such that

$$\left\|L_{n_1,n_2}(\rho)\right\|_\rho \leq M.$$

Theorem 3. (Gadžiev, 1980) Let $f_0 = e_{00}(t,s) = 1, f_1 = e_{10}(t,s) = t, f_2 = e_{01}(t,s) = s,\ f_3 = e_{20}(t,s) + e_{02}(t,s) = t^2 + s^2$ be the test functions of two variables and $\{L_{n_1,n_2}\}$ be any sequence of linear positive operators from C_ρ to B_ρ satisfying the following conditions:

$$\lim_{n_1,n_2\to\infty} \left\|L_{n_1,n_2}(f_i) - f_i\right\|_\rho = 0, i = 0,1,2,3, \tag{2}$$

Then for any function $f \in C_\rho^0$,

$$\lim_{n_1,n_2\to\infty} \left\|L_{n_1,n_2}(f) - f\right\|_\rho = 0,$$

And there exists a function $f^* \in C_\rho/C_\rho^0$, such that

$$\lim_{n_1,n_2\to\infty}\left\|L_{n_1,n_2}(f^*)-f^*\right\|_\rho \geq 1.$$

Now, we consider the following linear positive operators:

$$\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f;x,y)=\begin{cases}M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f;x,y), & (x,y)\in I\left(\alpha_{n_1},\beta_{n_2}\right)\\ f(x,y), & otherwise,\end{cases} \tag{3}$$

where $I\left(\alpha_{n_1},\beta_{n_2}\right)=\left\{(x,y):0\leq x\leq\alpha_{n_1},0\leq y\leq\beta_{n_2}\right\}$, $\left\{\alpha_{n_1}\right\}$ and $\left\{\beta_{n_2}\right\}$ are sequences of real numbers such that

$$\lim_{n_1\to\infty}\alpha_{n_1}=\infty=\lim_{n_2\to\infty}\beta_{n_2}.$$

We have the following weighted approximation result for the bivariate operators $\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$.

Theorem 4. Let $\left\{q_{1,n_1}\right\}$ and $\left\{q_{2,n_2}\right\}$ be any sequences such that $0<q_{1,n_1},q_{2,n_2}<1$ satisfying the condition given in (1) and $\left\{\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}\right\}$ be a sequence of linear positive operators defined by (3). Then for all functions $f\in C_\rho^0$, we have

$$\lim_{n_1,n_2\to\infty}\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f)-f\right\|_\rho=0.$$

Proof. We show that $\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$ is an operator from C_ρ to B_ρ. By using Lemma 2, we can write that

$$\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(\rho)\right\|_\rho=\sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+}\frac{\left|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(1+t^2+s^2;x,y)\right|}{1+x^2+y^2}$$

$$\leq 1+\sup_{(x,y)\in I\left(\alpha_{n_1},\beta_{n_2}\right)}\frac{\left|1+M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(t^2;x,y)+M_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(s^2;x,y)\right|}{1+x^2+y^2}$$

$$\leq 1+\sup_{(x,y)\in I\left(\alpha_{n_1},\beta_{n_2}\right)}\frac{1}{1+x^2+y^2}\left[1+\frac{[n_1]_{q_{1,n_1}}[n_1+1]_{q_{1,n_1}}x^2}{q_{1,n_1}^4[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}\right.$$

$$+\frac{[2]_{q_{1,n_1}}[n_1]_{q_{1,n_1}}x}{q_{1,n_1}^3[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}+\frac{[n_2]_{q_{2,n_2}}[n_2+1]_{q_{2,n_2}}y^2}{q_{2,n_2}^4[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}$$

$$+\frac{[2]_{q_{2,n_2}}[n_2]_{q_{2,n_2}}y}{q_{2,n_2}^3[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}\Bigg].$$

$$\leq \Bigg[1+\frac{[n_1]_{q_{1,n_1}}[n_1+1]_{q_{1,n_1}}}{q_{1,n_1}^4[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}+\frac{[2]_{q_{1,n_1}}[n_1]_{q_{1,n_1}}}{q_{1,n_1}^3[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}$$

$$+\frac{[n_2]_{q_{2,n_2}}[n_2+1]_{q_{2,n_2}}}{q_{2,n_2}^4[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}+\frac{[2]_{q_{2,n_2}}[n_2]_{q_{2,n_2}}}{q_{2,n_2}^3[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}\Bigg]$$

$$\leq 8.$$

Therefore, $\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}$ is an operator from C_ρ to B_ρ. It suffices to show that the conditions (2) of Theorem 3 are satisfied to complete the remain of the proof.

It is obvious that

$$\lim_{n_1,n_2\to\infty}\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_0;.)-f_0\right\|_\rho=0.$$

By considering Lemma 2, we can write

$$\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_1;.)-f_1\right\|_\rho=\sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+}\frac{\left|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(t;x,y)-x\right|}{1+x^2+y^2}$$

$$\leq\left|\frac{[n_1]_{q_{1,n_1}}}{q_{1,n_1}[n_1-2]_{q_{1,n_1}}}-1\right|\sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})}\frac{x}{1+x^2+y^2}$$

$$\leq\left|\frac{[n_1]_{q_{1,n_1}}}{q_{1,n_1}[n_1-2]_{q_{1,n_1}}}-1\right|. \tag{4}$$

By (4) and the condition (1), we obtain

$$\lim_{n_1,n_2\to\infty}\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_1;.)-f_1\right\|_\rho=0.$$

Similarly, we can get

$$\lim_{n_1,n_2\to\infty}\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_2;.)-f_2\right\|_\rho=0.$$

On the other hand, by Lemma 2, we can write

$$\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3;.)-f_3\right\|_\rho$$

$$=\sup_{(x,y)\in\mathbb{R}_+\times\mathbb{R}_+}\frac{\left|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(t^2+s^2;x,y)-(x^2+y^2)\right|}{1+x^2+y^2}$$

$$\leq \left|\frac{[n_1]_{q_{1,n_1}}[n_1+1]_{q_{1,n_1}}}{q_{1,n_1}^4[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}-1\right|\sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})}\frac{x^2}{1+x^2+y^2}$$

$$+\frac{[2]_{q_{1,n_1}}[n_1]_{q_{1,n_1}}}{q_{1,n_1}^3[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}\sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})}\frac{x}{1+x^2+y^2}$$

$$+\left|\frac{[n_2]_{q_{2,n_2}}[n_2+1]_{q_{2,n_2}}}{q_{2,n_2}^4[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}-1\right|\sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})}\frac{y^2}{1+x^2+y^2}$$

$$+\frac{[2]_{q_{2,n_2}}[n_2]_{q_{2,n_2}}}{q_{2,n_2}^3[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}\sup_{(x,y)\in I(\alpha_{n_1},\beta_{n_2})}\frac{y}{1+x^2+y^2}.$$

$$\leq \left|\frac{[n_1]_{q_{1,n_1}}[n_1+1]_{q_{1,n_1}}}{q_{1,n_1}^4[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}-1\right|+\frac{[2]_{q_{1,n_1}}[n_1]_{q_{1,n_1}}}{q_{1,n_1}^3[n_1-2]_{q_{1,n_1}}[n_1-3]_{q_{1,n_1}}}$$

$$+\left|\frac{[n_2]_{q_{2,n_2}}[n_2+1]_{q_{2,n_2}}}{q_{2,n_2}^4[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}}-1\right|+\frac{[2]_{q_{2,n_2}}[n_2]_{q_{2,n_2}}}{q_{2,n_2}^3[n_2-2]_{q_{2,n_2}}[n_2-3]_{q_{2,n_2}}},$$

which implies that

$$\lim_{n_1,n_2\to\infty}\left\|\widetilde{M}_{n_1,n_2}^{q_{1,n_1},q_{2,n_2}}(f_3)-f_3\right\|_\rho=0.$$

6. Conclusion

Let $\{q_{1,n_1}\}$ and $\{q_{2,n_2}\}$ be two sequences satisfying the condition (1). Under the condition (1), we obtained the uniform convergence theorem and the convergence theorem with respect to theweighted norm $\|\quad\|_\rho$.

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CHAPTER IV

BOILING WATER REACTORS

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1. Introduction

Boiling Water Reactors (BWR) where light water is used as cooler and moderator. BWR are nuclear power generation reactors that have been used in electricity generation for many years and are in constant development. The first experimental reactor for boiling water reactor (BORAX-1) was built in 1953 at Argonne National Laboratories, Idaho. After many experimental reactors designed over the years, Dresden-1, the first commercial BWR reactor type, was commissioned in California in 1961. Although these first models are called BWR-1, they are not considered as a complete BWR since they are steam boilers and steam generators. In the following years, BWRs have been transformed into simpler and more useful designs.

The first commercial BWRs were produced by the companies Allis-Chambers and General Electric (GE). Although Allis-Chambers designed reactors are not used today, GE designed BWRs have been developed and have survived to the present day. Other companies producing BWR reactors are ASEA-Atom, Kraftwerk Union (KWU), Hitachi and Toshiba. Today, BWR reactors are used commercially in Finland, Germany, India, Japan, Mexico, the Netherlands, Spain, Sweden, Switzerland and Taiwan.

The difference of BWRs from other light water reactors is that the steam is produced directly from the reactor core instead of steam generators or heat exchangers. This allows BWR reactors to operate with direct cycle. The essential components of a BWR reactor are pressure vessel, reactor guard, control systems and reactor safety systems. Today, BWRs have an output power of 570 to 1300 MWe.

First cycle; consists of core, turbine-generator, condenser, pre-heater and pumps. Since the water vapor produced in the first cycle of BWRs goes directly to the turbine and generator, the cooling water is also called "feed water".

Coolant water enters the reactor vessel containing nuclear fuel by pumps and evaporates by heating with the help of the nuclear energy generated. The temperature of the water entering and leaving the reactor vessel varies according to the reactor design. The entering water temperature is about 215 °C and the leaving steam temperature is around 288 °C (Nükleer Akademi, 2021).

As stated before, light water is used as moderator and coolant in BWR reactors. There is more light water than heavy water in nature. The steam obtained from light water rotates the turbines without the need for any steam generator, making the reactor design simple and inexpensive compared to other reactors.

Steam production and core cooling in boiling water reactors are provided by light water. This system consists of two water cycles. In the first cycle, the water in the embers evaporates by taking the heat energy and carries the energy to the turbine. In the second cycle, the water taken from the river or the sea and the steam coming out of the turbine in the primary cycle are turned into water again.

The water vapor formed reaches the turbine-generator building by passing through the moisture separators and steam dryers placed on the reactor vessel. Steam hitting the blades of the turbine causes the generation of electrical energy in the generator.

The steam sent from the turbine to the condenser located just below the turbine is converted to water and passed through the preheaters by means of pumps and transferred to the core to evaporate again.

In the first cooling system, there are three different types of pumps: feed water pumps, circulation pumps and jet pumps. The first of these; It is the "feed water" pump whose task is to send the cooling water from the turbine island to the reactor vessel. Latter; Circulation pumps placed on both sides of the reactor core whose task is to take some of the cooling water from the reactor vessel and return it to the reactor vessel in a rapid and pressurized manner. These motor driven pumps are used to control the reactor power. Thirdly, we can count the "jet" pumps inside the reactor vessel, which are also a part of the recirculation system. These pumps; It operates with the help of the suction force arising as a result of the pressure difference between the two nozzles of the pump "fast and pressurized water" sent by the circulation pump. In a normal BWR, there can be approximately 16-24 of these pumps. The purpose of jet pumps is to keep circulation pumps small and to increase their functionality.

Second water cycle; It consists of pump, condenser, cold water supply and related piping systems. The water vapor hits the turbine blades and loses energy and is sent to the condenser. Here, thanks to the cold water obtained from the sea or the river, it turns into a liquid again (Nükleer Akademi, 2021).

2. BWR Reactor Pressure Vessel and Core Structure

BWR pressure vessel; it is a pressure vessel made of low-alloy steel, 6.4 m in diameter, 22 m in height, 152 mm in thickness and internally covered with austenitic stainless steel (Dağlı, 2018). In the center of a typical BWR reactor pressure vessel, there are fuel bundles and control rods called cores as seen in Figure 1.

Moisture traps and steam dryers are located at the top of the reactor vessel. At the bottom of the pressure vessel are control guide tubes and control rod drivers that control the control rods.

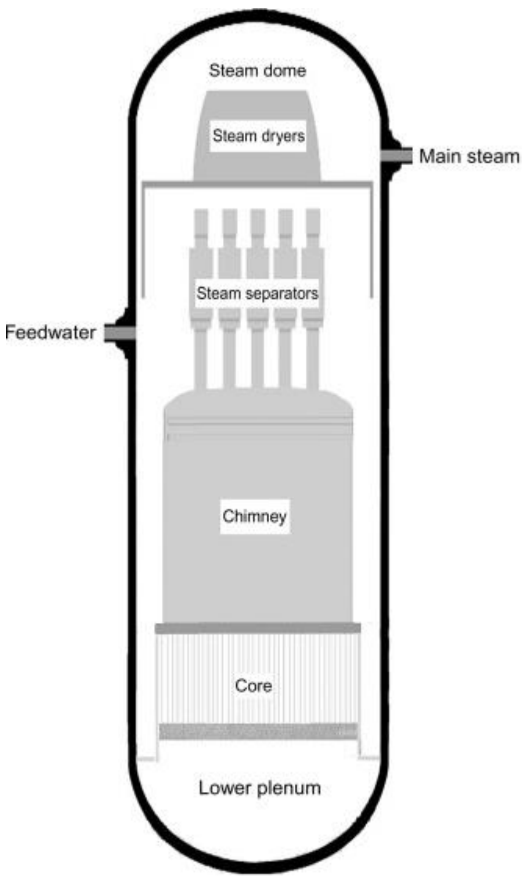

Figure 1. General Structure of BWR Core (Guerrero, A. P. and Paredes, G. E., 2019)

3. Fuel

UO_2 pellets containing approximately 3% enriched uranium are used in BWR reactors. Although the fuel rods are located in the reactor core in bundles such as 7x7, 8x8 or 9x9 in line with the design of the reactor, the most commonly used mesh system is the 8x8 array (Moore and Nots, 1989). In a BWR with an 8x8 fuel rod array, 62 fuel rods, a spacer water rod and a water rod are covered with Zr-2 armor and placed in an 8x8 tetragonal grid.

Although Zircaloy-2 is used as the armor material on the outer surface of the fuel rods; in 2013, GE company produced an armor material containing 1% Nb, 1% Sn and 0.35% Fe to prevent the deterioration of the fuel channel due to chemical interaction, and GE company currently uses this fuel armor in 8% of the cores it produces. On the other hand, Toshiba and ceramic company Ibis developed SiC as fuel armor. Its important advantages are that SiC can be easily used up to 1700 °C and the neutron absorption cross section is lower than conventional coatings.

The UO_2 pellets are placed in the armor material and sealed under He gas pressure. In order to keep the internal pressure of the fuel rods in balance during the fission reaction, there is a gas collection chamber where the gases formed as a result of the reaction can accumulate.

A typical BWR core contains 800 fuel elements containing about 140 tons of uranium. Each bundle of fuel remains in the reactor core for approximately 4.5 years. Once a year, BWR reactors are shut down for 4 to 6 weeks for fuel change and reactor maintenance. Different richness (low, medium, high) fuel rods are used in order to adjust the power level in the reactor core. This is possible when the fuel regeneration time comes, using the used fuel bundles that have not reached the maximum depletion value in different positions (Baltacıoğlu, 1995; Dağlı, 2018; Öztek and Kessler, 2003; World Nuclear Association, 2019).

4. Control Sticks

BWR control rods; Unlike other light water reactors, they are rods that enter from the lower part of the reactor vessel and consist of two cross-shaped wings in order to move between 4 fuel channels. An advantage for BWR reactors is that these rods comfortably grip the fuel assemblies. Control rods are made of materials where B_4C, Hf or both are used together according to their neutron absorbing feature. The lower part of the bar is designed like an umbrella in order to reduce the speed of the control rod in case of an accident.

Control rods are moved by hydraulic pressure drive or motor drive. Nitrogen gas is used as the driving force in both systems. When an abnormal situation occurs in the nuclear reactor that would require the reactor to be shut down, the control rods are quickly pushed towards the chord. In case the control rods do not fully enter the cora, the boric acid spray system is activated and the cora neutron absorbing material is sprayed.

In BWRs, fixed neutron poison called Gadolinia (Gd_2O_3) is also used to prevent the depletion of the fissionable material in the fuel and the accumulation of fission products while the reactor is active. Gd_2O_3 is mixed with UO_2 fuel pellets and used for long term power distribution control (World Nuclear Association, 2019).

The power of BWR reactors is controlled by control rods and motorized circulation pumps. By changing the flow rate of water with circulation pumps, neutrons can be slowed down, thus keeping the reactor power under control.

It takes a long time to calculate fuel assembly parameters in BWR reactors. In order to shorten this process and to provide symmetry between fuel bundles, a core cage design was made.

5. Security Systems

5.1. Emergency core cooling system

In cases such as a decrease in the water level in the reactor pressure vessel or loss of coolant, an emergency core cooling system is automatically included in the reactor and the reactor stops working. Position; to prevent fuel armor damage. The emergency core cooling system consists of a total of 5 systems: 3 low pressure core spraying, high pressure core cooling and low pressure core cooling.

5.2. Reactor guard building

Reactor protection building; It is a structure consisting of two parts, primary and secondary protection container. Position; It is to prevent the emission of radioactive materials left in the cooler under high pressure and temperature when a fuel failure occurs. The primary containment vessel (dry well) includes the reactor pressure vessel and the pressure relief system, while the secondary containment vessel covers the reactor building and the primary containment vessel.

The pressure prevention pool located in the primary protection container reduces the pressure by condensing the vapor in the protection container in case of an accident.

While Mark I was used in BWR-2, BWR-3 and BWR-4, Mark III design was used in BWR-6 (Dağlı, 2018).

Steel lead alloys and heavy concrete are used in the structure of the guard building. Concrete content consists of materials such as limonite

($Fe_2O_3H_2O$), barite (barium sulphate $BaSO_4$), magnetite (Fe_3O_4). The density of the heavy concretes produced is higher than 2.600 kg/dm^3 (Çapalı, 2015).

6. Result

Rapid population growth, industrialization and widespread use of technology in the world, inability to meet the energy power of fossil fuels with the increasing demands and needs today it increases the need for energy. Accordingly, the increasing energy in the world With its need, fossil (non-renewable) energy resources are rapidly being used up. Today approximately 87% of electrical energy production is petroleum, coal, which are limited in nature, from fossil energy sources such as natural gas and renewable energy sources are being. Rapid consumption of fossil energy resources and environmental problems their existence increases the search for alternative energy sources. Sun, wind, stream, geothermal renewable energy sources that do not cause environmental problems such as although there are resources that can be filled, the amount of energy obtained from these sources can not meet the need.

The most important resource that can be a solution to countries' search for alternative energy is nuclear energy. In nuclear reactors, nuclear energy is produced by fission or fusion. Both the high degree of energy released as a result of the reaction is transferred out of the reactors in various ways. it is converted into necessary energy. Higher than fission and fusion reactions obtaining energy, avoiding resource problems, however necessary security it will not cause environmental problems when necessary precautions are taken, it shows that it will be obtained from nuclear reactors. Only fission reactors today used in energy production. As of 2020, the world's electrical energy need 454 fission reactors that meet approximately 13% are actively working.

Today's fission reactors in use contain a very small percentage of natural uranium (1%) is used and the rest remains as waste. Therefore from natural uranium that the fissile fuel requirement being met will become unsatisfactory expected.

The lifespan of nuclear power plants has been extended from 30 to 40 years to an average of 60 years by using various technologies. One of the reasons why developed countries that have a say in the world do not need new reactors is that they renew and use the existing nuclear power plant. This is not the case for wind and solar power plants. These power plants have a lifespan of 20-30 years.

Today, while the energy needs of many countries are met by the energy produced in fission reactors, studies on the fusion reaction are continuing theoretically and experimentally.

Most of the fission reactors in the world are light water reactors (LWR). This is because normal water is used as a coolant and retarder in the reactor. In addition to the fact that it is cheaper and easily accessible compared to the alternatives of normal water, it reduces the density of steam formation, providing neutron balance.

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CHAPTER V

LIQUID CHROMATOGRAPHIC METHODS USED FOR DETERMINATION OF HONEY PROPERTIES

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1. Introduction

Honey is a natural sweetener made by honey bees and obtained from floral nectar (Karabagias, Badeka, Kontakos, Karabournioti, & Kontominas, 2014). Honey is a complex substance that contains about 200 compounds including complex mixture of carbohydrates as well as small amounts of other components such as phenolic acids, organic acids, minerals, vitamins, enzymes, flavonoids and proteins. These compounds, which include phenols, carotenoids, flavonoids, enzymatic and non-enzymatic chemicals are considered antioxidants (Bueno-Costa et al., 2016; Hunter et al., 2021). Moreover, it is regarded as a crucial component of traditional medicine (Ilia, Simulescu, Merghes, & Varan, 2021; Ramanauskiene, Stelmakiene, Briedis, Ivanauskas, & Jakštas, 2012; Seraglio et al., 2021).

Honey is a natural sweet substance made by honey bees from plant nectar, secretions of live plant parts, or excretions of plant sucking insects on alive plant parts, according to the Codex Alimentarius. Honey collected by bees is reformated by combining it with their own unique ingredients, then deposited, dehydrated, stored, and allowed to develop and mature in the honey comb. (Alimentarius, 2001).

Honey is a concentrated solution of fructose (Fr), glucose (Gl), and sucrose (Sc) as well as a few additional disaccharides and oligosaccharides found in low concentrations (Escuredo, Dobre, Fernández-González, & Seijo, 2014).

Adding economic sweeteners (cane sugar/refined beet sugar/corn syrup, etc.) to honey and feeding honeybees with Sc are the most common methods of honey tampering (Puscas, Hosu, & Cimpoiu, 2013). Botanical and geographical source of the honey product must be mentioned on the packaging label, according to EU honey regulation 110/2001 (Council Directive, 2001). These rules are designed to guarantee product quality and authenticity, as well as to protect customers from product adulteration and fraud. So its quality must be checked by analytical methods in order to ensure genuity and protect consumers from commercial speculation. For this purpose, it is very crucial that the using chromatographic methods assessment of honey quality control in honey products.

2. General Properties and Quality Standards of Honey

Honey is a supersaturated sugar solution with a preponderantly Fr (~38%) and Gl (~31%) content (Pascual-Maté, Osés, Marcazzan, et al., 2018). Sugars represent the majority part of honey composition (solids part of 95-99%). The most abundant sugars are Fr and Gl in honey but saccharose, maltose, trehalose, and elizitose are also commonly mentioned. Its composition is strongly affected by the sorts of flowers used by the bees, regional and climatic variables (Mendes, Proença, Ferreira, & Ferreira, 1998). About 75% of the sugars in honey are monosaccharides. Honey's sugars are responsible for features like viscosity, energy value, granulation, hygroscopicity (Kamal & Klein, 2011). The amount of saccharose in the honey is crucial for detecting feeding bees with sugar or adulteration by direct addition of sugar and syrups (high Fr corn syrupe) (Anklam, 1998). Fr to Gl ratio could be used to characterize honey products from various origins. Furthermore, it may show the tendency of honey to crystallize (Abu-Tarboush, Al-Kahtani, & El-Sarrage, 1993). The Fr-to-Gl ratio is usually about 1.2:1.0, while there is some difference depending on the honey floral origin (da Silva, Gauche, Gonzaga, Costa, & Fett, 2016; Escuredo et al., 2014). Honeys with a Fr/Gl ratio of 0.99 to 1.20 include citrus, heather, eucalyptus sunflower, and dandelion. Other honeys with a ratio of 1.30 to 1.40 include rhododendron, thyme, and honeydew, whereas acacia and chestnut honeys have a ratio of 1.59 to 1.67, indicating a clear abundance of Fr when compared to Gl (Baglio, 2018).

Proline is the most plentiful amino acid in the pollen and honey (Iglesias et al., 2006). Moreover, glutamic acid, aspartic acid, glutamine, glycine, arginine, tyrosine, histidine, leucine, phenylalanine, asparagine tryptophan and alanine etc. are found in honey (María Luz Sanz, Del Castillo, Corzo, & Olano, 2003). The amount of protein in honeybees varies depending on their origin. (Lee, Lee, Cha, Choi, & Rhee, 1998).

Honey is generally containing 17-18% of water but with easy fermentation methods, the aqueous component can reach up to %21 (Baglio, 2018). The water content of honey is considered one of the most crucial features since it influences viscosity, specific weight, maturity, flavor, and crystallization (Al-Farsi et al., 2018).

Honey includes trace levels of vitamins, particularly the vitamin B complex; B1, B2, B3, B5, B6, B8 (or H), B9 and vitamin C is also present. Since the honey has a low pH, those vitamins present in honey are preserved (Ciulu et al., 2011; León-Ruiz, Vera, González-Porto, & San Andrés, 2013).

Phenolic substances are secondary plant metabolites with a specific phenolic pattern that can be used to distinguish honey kinds based on their botanical origin. Phenolic substances are divided into two categories: flavonoids and phenolic acids. (Mădaş et al., 2020).

Honey contains trace elements and a wide variety of minerals that are found in concentrations ranging from 0.1% to 1.0%. Honeydew honeys contain more minerals than nectar honeys, resulting in higher electrolytic conductivity. The most abundant metal is potassium, which is followed by Ca, Mg, Na, S and P. Fe, Cu, Zn, and Mn are examples of trace elements (Lachman et al., 2007; Liu et al., 2021).

Weather conditions, processing, manipulation, packaging, and storage length, as well as the geographical areas, flowers, climate, and honeybee species involved in its production, all influence the composition, color, aroma, and flavour of honey (da Silva et al., 2016; Escuredo et al., 2014).

Due to the presence of approximately 0.57 organic acid, all honeys have a slight acidity value. These organic acids are produced by enzymes released by honeybees from carbohydrates. These acids are responsible for the colour and flavor of honey, as well as its chemical features such as electrical conductivity, pH and acidity (da Silva et al., 2016).

The acid-catalyzed dehydration of hexoses produces hydroxymethylfurfural (HMF). Its content changes based on pH and heating or storage conditions, and it's utilized as a freshness indicator for honey (Valle Morales, Sanz, Martín-Álvarez, & Corzo, 2009).

Generally, various parameters are used to assess the quality of honey, including moisture content, free acidity, pH, concentration of organic acid, and HMF content (Serra Bonvehí, Bentabol Manzanares, & Santos Vilar, 2004).

3. Chromatographic Methods Used in Honey Quality Assessment

Liquid chromatography (LC) is a modern technique that is quite sensitive, accurate, rapid and precise using wide range of monitoring properties of many food products. (Ravisankar, Navya, Pravallika, & Sri, 2015; Vogeser & Seger, 2008). LC is today one of the most important analytical techniques derived from classical column chromatography. LC is one of the popular analytical techniques derived from classical column chromatography. LC involves the separation of a sample into its individual parts. This separation occurs as a result of sample’s interactions with the mobile and stationary phases (Gupta, Jain, Gill, & Guptan, 2012). Studies using LC methods to determine honey components such as sugars, HMF, vitamins and phenolic compounds are summarized in the **Table 1.**

Table 1. Applied liquid chromatographic methods for determination of properties of honeys.

Compounds	Analytical Methods	References
Sugars	HPLC-ELSD, HPAEC-PED, HPAEC-PAD, HPLC-RID, HPLC-PDA	(Corradini, Cavazza, & Bignardi, 2012;Da Costa Leite et al., 2000; de la Fuente, Ruiz-Matute,Valencia-Barrera, Sanz, & Martínez Castro, 2011;de la Fuente, Sanz, Martínez-Castro, Sanz, & Ruiz-Matute,2007; Escuredo et al., 2014; V. Morales, Corzo, & Sanz, 2008; Pereira Da Costa & Conte-Junior, 2015; Qiangsheng, Qiyan, Kun, & Chunlin, 2013; M. L. Sanz, Gonzalez, de Lorenzo, Sanz, & Martínez-Castro, 2005; Sousa et al., 2016; Zhou et al., 2014)
HMF	HPLC-RID, HPLC-PDA, HPLC-UV	(Alghamdi et al., 2020; Ghramh, Khan, Zubair, & Ansari, 2020; Lemos, Santos, & Santos, 2010; Makawi et al., 2009; Spano et al., 2006; Szczesna & Rybak-Chmielewska, 1999; Zappalà, Fallico, Arena, & Verzera, 2005)

Table 2. ***continued***

Vitamins	HPLC-UV, HPLC-PDA	(Bonta et al., 2013; Ciulu et al., 2011; León-Ruiz et al., 2013; León-Ruiz, Vera, González-Porto, & Andrés, 2011; Ragab, El-Yazbi, & El-Hawiet, 2020; Santana et al., 2021; Sawicki, Bączek, & Starowicz, 2020)
Phenolic compounds	HPLC-UV, HPLC-PDA	(Campone et al., 2014; Kenjerić, Mandić, Primorac, Bubalo, & Perl, 2007; Pascual-Maté, Osés, Fernández-Muiño, & Sancho, 2018; Yao et al., 2004)

HPAEC-PED: High Performance Anion-Exchange Chromatography Coupled-Pulsed Electrochemical Detection
HPAEC-PAD: High Performance Anion-Exchange Chromatography Coupled-Pulsed Amperometric Detection
ELSD: Evaporative Light Scattering Detection
RID: Refractive Index Detection
PDA: Photo Diode Array Detection
UV: Ultraviolet Detection

4. Conclusion

Quality control step is important to determine the origin of honey, honey labeling and also it is crucial to protect consumers or producers from adulteration and fraud. For this purpose, several analytical techniques are used to characterize the honey. Especially, the application of HPLC technology to honey analysis has been increased day by day. HPLC is used as a separation technique in honey composition analysis due to rapid analysis, separation efficiency, enabling low detection and quantification limits and cost-effective processes.

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CHAPTER VI

PALLADIUM-CATALYZED SUZUKI CROSS COUPLING REACTIONS WITH BENZIMIDAZOLIUM SALTS

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1. Introduction

Benzimidazole is an aromatic heterocyclic compound (Figure 1). Benzimidazole derivatives are of importance because of their various curative applications (Yerragunta et al., 2014). Benzimidazolium salts have been frequently studied as they serve in the synthesis of N-heterocyclic carbene (NHC) complexes (Riaz et al., 2020; Scattolin et al., 2021; Atif et al., 2020; Haziz et al, 2020). NHC complexes have exhibited high catalytic efficiencies in various organic reactions (Liang and Song, 2020; Wang et al., 2020; Demir Atli, 2020).

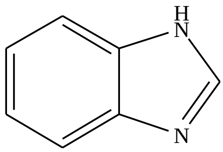


Figure 1. Benzimidazole

Cross coupling reactions which carbon-carbon bond formations occur are of great importance in both synthetical chemistry and in industry (Biajoli et al., 2014). Suzuki cross coupling reactions actualise in the basic medium and by using an organoboron compound as a nucleophile (Scheme 1). For this reaction, firstly catalyzed by palladium (Miyaura et al., 1979; Miyaura and Suzuki, 1979; Pagett and Lloyd-Jones, 2019), the catalysts containing different metals have also been employed (Sonei et al., 2021; Zhang et al., 2021; Hajipour and Malek, 2021).

$$\text{R-X} + \text{R'-BY}_2 \xrightarrow{\text{base}} \text{R-R'}$$

Scheme 1. Suzuki Cross-Coupling Reaction

For Suzuki cross coupling reactions, many NHC-palladium complexes have been utilised as effective catalysts (Chen and Kao, 2017; Wang et al., 2018; Kaloğlu and Özdemir, 2019, Zhang et al. 2020) . In addition, in situ formed catalytic systems composed of $Pd(OAc)_2$, benzimidazolium salt and a base have been successfully employed in Suzuki cross coupling reactions (Akkoc and Gok, 2013; Özdemir et al., 2004; Lin et al., 2017, Wang et al., 2021, Akkoç, 2019).

In this work, synthesis of 1-(2,5-dimethylbenzyl)-5,6-dimethyl-benzimidazole and four benzimidazolium derivatives are reported. All the synthesized compounds are not listed in the literature. The structural characterizations are performed by spectroscopic methods and elemental analysis. The catalytic activities of the mixtures of the benzimidazolium salts and $Pd(OAc)_2$ in Suzuki cross coupling reactions of some aryl chlorides with phenylboronic acid are tested.

2.Experimental

2.1. Materials and Measurements

All chemicals were utilised as purchased. The experiments were carried out in air. NMR, IR and elemental analyses were fulfilled by a Varian VNMRJ spectrometer, Perkin-Elmer FT-IR spectrometer and a LECO-932 CHNS model apparatus, respectively.

2.2. Preparation of 1-(2,5-dimethylbenzyl)-5,6-dimethylbenzimidazole (1)

Potassium hydroxide (0.51 g, 9 mmol) was added to ethanol (15 mL) solution of 5,6-dimethylbenzimidazole (0.73 g, 5 mmol). The suspension was stirred at 25 °C for 2 hours. 2,5-dimethylbenzyl chloride (0.89 mL, 6 mmol) was added and it was refluxed overnight. After water (50 mL) addition, extraction with CH_2Cl_2 (3x15 mL), drying with Na_2SO_4, filtration and removing of CH_2Cl_2 processes, the product was recrystallized from toluene/n-hexane mixture at +4 °C. Yield: 1.18 g, 89%. m.p.: 118-119 °C. ^1H NMR (400 MHz, chloroform-*d*): δ = 7.68 (s, 1H, ArH), 7.60 (s, 1H, ArH), 7.15 – 7.03 (m, 3H, ArH), 6.73 (s, 1H, ArH), 5.21 (s, 2H, NCH_2), 2.38 (s, 3H, CH_3), 2.36 (s, 3H, CH_3), 2.26 (s, 3H, CH_3), 2.23 (s, 3H, CH_3) ppm. ^{13}C NMR (101 MHz, chloroform-*d*) δ = 142.48, 142.40, 136.18, 133.22, 132.76, 132.72, 132.18, 131.12, 130.67, 129.00, 128.43, 109.97, 46.79, 20.98, 20.63, 20.31, 18.63 ppm. FT-IR : ν_{NCN} = 1495 cm^{-1}. Anal. Calc. for $C_{18}H_{20}N_2$ (M_w = 264.40 g/mol): C, 81.76; H, 7.64; N, 10.60. Found: C, 81.05; H, 7.32; N, 10.63.

2.3. General Method for Preparation of the Salts

The mixture of **1** (1 mmol) and alkyl halide (1.1 mmol) in dimethylformamide (3 mL) was stirred at 70 °C for 24 hours. The solution was cooled. White solids were precipitated by adding excess diethyl ether. Recrystallization was practised from methanol/diethyl ether (1/4) at -18°C.

2.3.1. 1-(2,5-dimethylbenzyl)-3-ethyl-5,6-dimethylbenzimidazolium Bromide (2)

Yield: 53%. m.p.: 194-195 °C. IR $\nu_{(CN)}$: 1552 cm^{-1}. ^{1}H NMR (400 MHz, chloroform-*d*) δ = 11.04 (s, 1H, NCHN), 7.44 (s, 1H, ArH), 7.13 (s, 1H, ArH), 7.08 – 7.00 (m, 2H, ArH), 6.87 (s, 1H, ArH), 5.71 (s, 2H, NCH_2), 4.61 (q, *J* = 7.3 Hz, 2H, $N\underline{CH_2}CH_3$), 2.39 (s, 3H, CH_3), 2.30 (s, 6H, CH_3), 2.21 (s, 3H, CH_3), 1.68 (t, *J* = 7.3 Hz, 3H, $NCH_2\underline{CH_3}$) ppm. ^{13}C NMR (101 MHz, chloroform-*d*) δ = 141.49, 137.34, 136.43, 133.09, 131.10, 130.47, 130.06, 129.87, 129.70, 128.67, 113.35, 112.67, 49.51, 42.85, 20.93, 20.72, 20.65, 19.10, 14.92 ppm. Anal. Calc. for $C_{20}H_{25}N_2Br$ (M_w = 373.37 g/mol): C, 64.33; H, 6.76; N 7.50. Found: C, 63.17; H, 6.66 ; N, 8.06%.

2.3.2. 1-(2,5-dimethylbenzyl)-3-(2-hydroxyethyl)-5,6-dimethylbenzimidazolium Bromide (3)

Yield: 62%. m.p.: 197-198 °C. IR $\nu_{(CN)}$: 1559 cm^{-1}. ^{1}H NMR (400 MHz, chloroform-*d*) δ = 10.10 (s, 1H, NCHN), 7.51 (s, 1H, ArH), 7.18 (s, 1H, ArH), 7.06 (q, *J* = 8.2 Hz, 2H, ArH), 6.95 (s, 1H, ArH), 5.59 (s, 2H, NCH_2), 4.97 (s, 1H, OH), 4.68 (t, *J* = 4.9 Hz, 2H, $NCH_2\underline{CH_2}OH$), 4.05 (t, *J* = 4.9 Hz, 2H, $N\underline{CH_2}CH_2OH$), 2.38 (s, 3H, CH_3), 2.32 (s, 3H, CH_3), 2.28 (s, 3H, CH_3), 2.23 (s, 3H, CH_3) ppm. ^{13}C NMR ^{13}C NMR (101 MHz, chloroform-*d*) δ = 141.51, 137.31, 137.22, 136.59, 133.26, 131.15, 130.24, 130.18, 130.05, 129.95, 129.16, 113.07, 112.93, 58.89, 49.60, 49.51, 20.90, 20.69, 20.60, 18.98.ppm. Anal. Calc. for $C_{20}H_{25}N_2OBr$ (M_w = 389.37 g/mol): C, 61.69; H, 6.48; N 7.20. Found: C, 60.36; H, 6.17; N, 7.28%.

2.3.3. 1-(2,5-dimethylbenzyl)-3-benzyl-5,6-dimethylbenzimidazolium Bromide (4)

Yield: 85%. m.p.: 208-209 °C. IR $\nu_{(CN)}$: 1559 cm^{-1}. ^{1}H NMR (400 MHz, chloroform-*d*) δ = 11.35 (s, 1H, NCHN), 7.45 (dd, *J* = 7.8, 1.7 Hz, 2H, ArH), 7.36 – 7.28 (m, 4H, ArH), 7.12 (s, 1H, ArH), 7.05 (q, *J* = 9.5, 8.6 Hz, 2H, ArH), 6.86 (s, 1H, ArH), 5.82 (s, 2H, NCH_2), 5.71 (s, 2H, NCH_2), 2.30 (s, 6H, CH_3), 2.26 (s, 3H, CH_3), 2.21 (s, 3H, CH_3) ppm. ^{13}C NMR (101 MHz, chloroform-*d*) δ = 142.10, 137.42, 137.38, 136.47, 133.12, 132.96, 131.16, 130.31, 130.08, 129.94, 129.85, 129.29, 129.08, 128.66, 128.09, 113.27, 51.18, 49.68, 20.91, 20.65, 19.04 ppm. Anal. Calc. for $C_{25}H_{27}N_2Br$ (M_w = 435.44 g/mol): C, 68.95; H, 6.26; N 6.44. Found: C, 68.02; H, 6.15; N, 6.60%.

2.3.4. 1,3-bis(2,5-dimethylbenzyl)-5,6-dimethylbenzimidazolium Chloride (5)

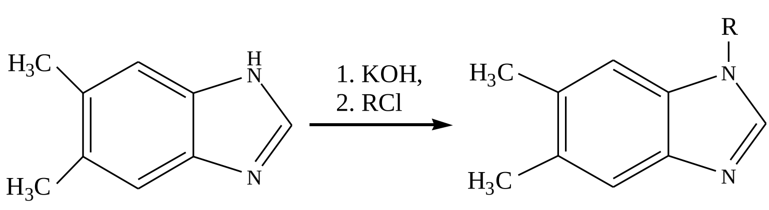
Yield: 74%. m.p.: 238-239 °C. IR $\nu_{(CN)}$: 1552 cm^{-1}. ^{1}H NMR (400 MHz, chloroform-*d*) δ = 11.62 (s, 1H, NCHN), 7.11 – 7.00 (m, 6H, ArH), 6.78 (s, 2H, ArH), 5.78 (s, 4H, NCH_2), 2.33 (s, 6H, CH_3), 2.26 (s, 6H, CH_3), 2.20 (s, 6H, CH_3) ppm. ^{13}C NMR (101 MHz, chloroform-*d*) δ = 143.53, 137.23, 136.31, 133.02, 131.12, 130.63, 130.09, 129.76, 128.23, 113.33, 49.63, 20.93, 20.67, 18.98 ppm. Anal. Calc. for $C_{27}H_{31}N_2Cl$ (M_w = 419.05 g/mol): C, 77.38; H, 7.47; N 6.69. Found: C, 76.92; H, 7.28; N, 6.50%.

2.4. General Method for Suzuki Coupling Reaction

$Pd(OAc)_2$ (0.01 mmol), the benzimidazolium salt (0.02 mmol), aryl chloride (1 mmol), phenylboronic acid (1.5 mmol), K_2CO_3 (1.2 mmol), dimethylformamide (3 mL) and water (3 mL) were added to a flask. The mixture was stirred at 80 °C for 2 hours. The the mixture was cooled. Water (20 mL) was added. Extraction with CH_2Cl_2 (3x10 mL) and drying with Na_2SO_4 were carried out. Filtration, removing the solvent and column chromatography gave the pure cross coupling product.

3. Results and Discussion

1-(2,5-dimethylbenzyl)-5,6-dimethylbenzimidazole compound **(1)** was prepared in high yield by deprotonation of 5,6-dimethylbenzimidazole with KOH and followed by N-alkylation with 1.2 equivalent of 2,5-dimethylbenzyl chloride in ethanol medium (Scheme 2). It is soluble in ethanol, dimethylformamide, toluene, methanol, ethyl acetate, tetrahydrofuran and dichloromethane; insoluble in n-pentane, n-hexane and water.

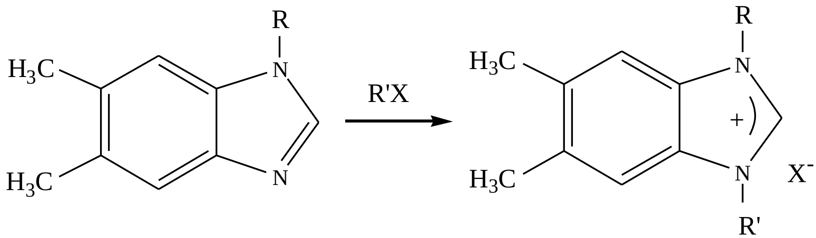


1

R = $CH_2C_6H_3(CH_3)_2$-2,5

Scheme 2. Synthesis of **1**

The reaction of **1** with 1.1 equivalent of alkyl halides in dimethylformamide at 70 °C gave the expected benzimidazolium salts (Scheme 3). The white solids were obtained in good yields. These salts are soluble in water, dichloromethane, dimethylformamide, ethanol and methanol; insoluble in diethyl ether and n-hexane.



1

2-5

R = $CH_2C_6H_3(CH_3)_2$-2,5

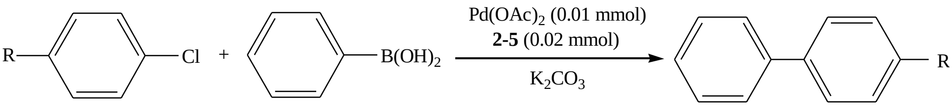
2: R' = CH_2CH_3 X = Br
3: R' = CH_2CH_2OH X = Br
4: R' = $CH_2C_6H_5$ X = Br
5: R' = $CH_2C_6H_3(CH_3)_2$-2,5 X = Cl

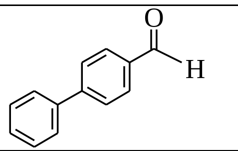
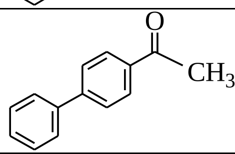
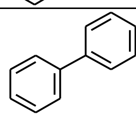
Scheme 3. Synthesis of **2-5**

The synthesized compounds are characterized by NMR and IR spectroscopic methods and also elemental analysis. NMR spectra of **1** affirm the structure of the compound. The signals seen at 11.04, 10.10, 11.35, and 11.62 ppm in ^{1}H NMR spectra of **2-5**, respectively, are attributed to acidic C2 protons. In ^{13}C NMR spectra of them, C2 atoms give resonances at 141.49, 141.51, 142.10 and 143.53 ppm, respectively. ν(C=N) peaks of **2-5** are found in the range of 1552-1559 cm^{-1} in the IR spectra. These spectroscopic values are similar to the formerly reported (Demir Atli, 2020; Şahin et al., 2020).

The benzimidazolium salts **2-5** were used coupled with $Pd(OAc)_2$ in Suzuki cross coupling reactions. K_2CO_3 was employed as base. The reactions of 4-chlorobenzaldehyde, 4′-chloroacetophenone and chlorobenzene with phenylboronic acid were carried through in dimethylformamide-water (1-1) at 80°C for 2 hours. After extraction process, the crude product was purified by column chromatography. The biphenyls were analysed by ^{1}H NMR spectroscopy (Figures 2-4). The isolated yields were noted (Table 1). As seen from the results, the catalytic systems have similar activities for each chloroarene. 4-acetylbiphenyl and 4-phenylbenzaldehyde compounds were obtained in good yields (Table 1, entries 1-8). Whereas biphenyl yields were low (Table 1, entries 9-12). These results indicate that aryl chlorides with electron withdrawing groups incline to high activity as found in the literature (Prajapati et al., 2020).

Table 1. The Catalytic Test Results



Entry	**R**	**Salt**	**Product**	**Yield (%)**
1	CHO	2		75
2		3		73
3		4		76
4		5		81
5	$COCH_3$	2		83
6		3		85
7		4		86
8		5		90
9	H	2		20
10		3		25
11		4		38
12		5		16

Conditions: Chloroarene (1 mmol), $Pd(OAc)_2$, $PhB(OH)_2$ (1.5 mmol), K_2CO_3 (2 mmol), **2-5**, dimethylformamide-water (3 mL-3 mL), 80 °C, 2 hours.

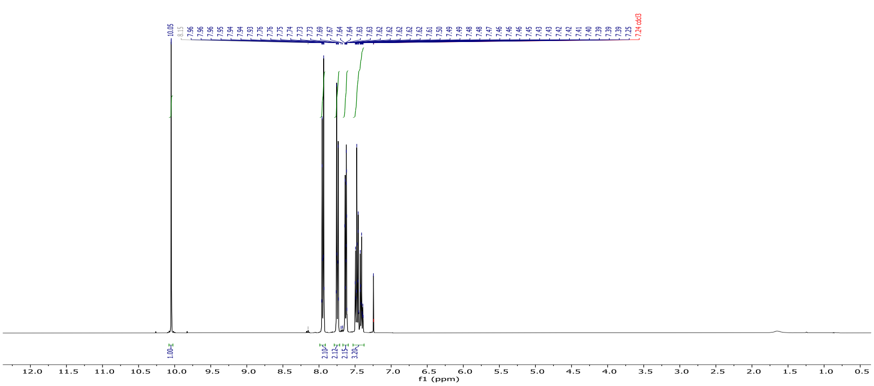


Figure 2. ^{1}H NMR Spectrum of 4-phenylbenzaldehyde

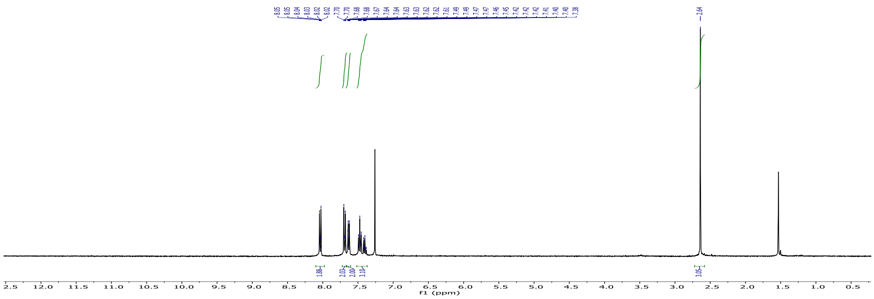


Figure 3. [1]H NMR Spectrum of 4-acetylbiphenyl

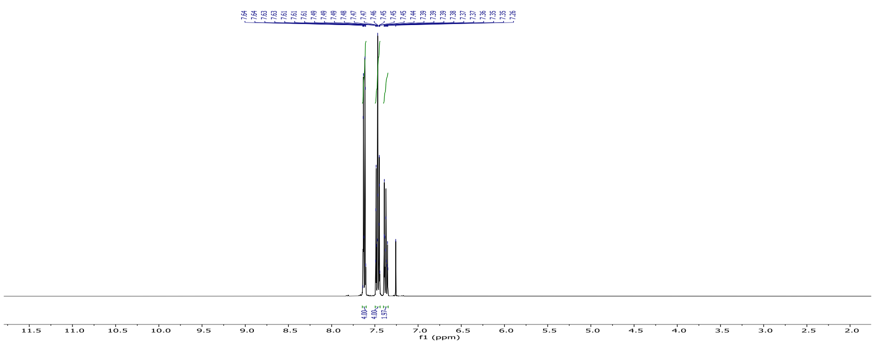


Figure 4. [1]H NMR Spectrum of Biphenyl

4. Conclusion

In this study, 1-(2,5-dimethylbenzyl)-5,6-dimethylbenzimidazole and four new benzimidazolium salts were prepared and characterized. It was deduced that the salts together with $Pd(OAc)_2$ showed catalytic activity in Suzuki cross-coupling reactions.

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CHAPTER VII

TRACE ELEMENTS IN METABOLISM AND THEIR FUNCTIONS

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1. Introduction

20 of the elements in the periodic table have functional and structural importance in the living body (Quigg, 2008). Approximately 98% of human metabolism consists of the elements phosphorus (P), oxygen (O), nitrogen (N), calcium (Ca), carbon (C), hydrogen (H). In addition, sulfur (S), magnesium (Mg), chlorine (Cl), sodium (Na), silicon (Si) and potassium (K) elements are also involved in metabolism (Aras &Ataman, 2006). It is important to take a certain amount of some inorganic substances such as molybdenum (Mo), manganese (Mn), cobalt (Co), iron (Fe), iodine (I), chromium (Cr), selenium (Se), copper (Cu) and zinc (Zn) for a healthy metabolism (Arcasoy, 2002). Elements found in ppm (µg/mL) and ppb (µg/L) concentrations in human metabolism are called trace elements (Sonmez, 2010). Elements that have various functions in human metabolism are divided into two groups as macro and trace elements. It is defined as a macro element, especially if the amount in metabolism is more than 100 mg⁄kg, and as a trace element if it is less. As another definition; elements with concentrations of g/L, g/kg in tissues and body fluids are called major (macro), elements with mg/L and mg/kg are called trace (minor) elements (Fraga, 2005). The elements in the content of the nutrients that enter the living metabolism are as follows:

- Major elements; Ca, P, K, Na, Cl, Mg
- Minor (trace) elements; Fe, Zn, Cu, Mn, I, Mo, Se, Cr (Sevinc et al., 2004).

2. Functions of Trace Elements

Trace elements have many functions in metabolism. These elements are essential for the biological, chemical and molecular functions of the cell in metabolism. They act as cofactors for many enzymes. They are also responsible for stabilizing the structures of enzymes and proteins. They bind to molecules in the receptor region of the cell membrane to control important biological processes. In addition, some trace elements are vital in redox reactions responsible for the production and use of metabolic energy (Lingamaneni et al., 2015).

These elements play a role in the stability of important biological molecules (Douglas et al., 2007). They are especially necessary for development, muscle and nerve functions, growth, normal cellular functions and the production of some hormones and connective tissue (Al-Fartusie & Mohssan, 2017).

In addition to organic molecules (protein, carbohydrate and fat), these trace elements are also needed for cell proliferation, replication and differentiation in living things (Zheng et al., 2008).

In particular, trace elements act as antioxidants in scavenging and neutralizing free radicals in the body (Parsons & Barbosa, 2007). Many important biological functions in humans can be affected in trace (trace) element deficiency (Farkhutdinova et al., 2006). Especially if the concentrations of these elements increase, they damage the metabolism (Ansari et al., 2004; Saracoglu et al., 2007).

3. Main Trace Elements And Their Duties

3.1. Iron (Fe)

Iron is abundant in the earth's crust and is largely obtained from the plant kingdom. This element is around 3-5 grams in the body, of which 75% is found in the blood and the rest in the liver, bone marrow and muscles. Iron-related enzymes are cytochrome A, B, C, F 450, cytochrome C reductase, catalases, peroxidases, xanthine oxidases, tryptophan pyrolase, succinate dehydrogenase, glucose 6 phosphate dehydrogenase, and choline, while Hb is found in myoglobin, cytochrome. The average daily iron requirement in metabolism is around 1-2 mg. The human body has to provide 20 mg of iron with external foods (Vasudevan & Sreekumari, 2007).

Deficiency of this trace metal leads to iron deficiency anemia. This deficiency leads to diseases such as fatigue, impaired attention, irritability and reduced memory (Lieu et al., 2001). In addition, iron deficiency anemia also leads to heart failure (Gil & Ferreira, 2014). When the amount of this element in metabolism increases, nausea, vomiting, diarrhea occur with liver damage. It also causes important diseases such as liver failure, diabetes, and arthritis (Andrews, 1999). Recently, it has been established that iron may play a role in esophageal carcinogenesis (Boult et al., 2008).

3.2. Zinc (Zn)

This element is the second most common element in human metabolism after iron. Zinc is needed for many enzyme activities in the human body. Enzymes containing zinc element are superoxide dismutase, retinene reductase, DNA and RNA polymerase, carboxypeptidase carbonic anhydrase, alkaline phosphatase (deoxyribonucleic acid, ribonucleic acid) thymidine kinase and alcohol dehydrogenase (Onat et al., 2006).

This element is active in many cellular events and is an essential trace element. In addition to being essential for nutrition, zinc has important functions as a cofactor of many enzymes and transcription factors. Zinc element is a biological catalyst and has important roles in activating many enzymes and hormones. Especially, this element is used in the secretion of hormones from the pituitary gland, the development of the fetus, the continuity of the immune system, the perception of taste and smell, the healing of wounds, during pregnancy, in keeping the plasma vitamin A level at normal levels, in the formation of sperm, in allergenic conditions against viruses and bacteria, plays an important role in the growth and development of the body during infancy and childhood. In addition, zinc protects the cell membranes against oxidative damage and ensures the continuity of the cell (Taskapan et al., 2007). In addition, zinc element is involved in the growth and replication of cells in metabolism (Belgemen & Akar, 2004).

3.3. Copper (Cu)

Copper element is a structural component of proteins and enzymes such as δ-aminolevulinate dehydratase, galactose oxidase, ascorbate oxidase, tyrosinase, ceruloplasmin, cytochrome C oxidase, lysyl oxidase, uricase, dopamine beta hydroxylase and superoxide dismutase. There is around 100–120 mg of copper in an adult human body (Unaldi & Yontem, 2011, Adam & Yigitoglu, 2012).

This element is involved in iron metabolism in the body. In particular, it contributes to the iron element in the synthesis of hemoglobin. It also plays a role in protein metabolism and improvement process (Tokman, 2007).

The deficiency of this element and the prolongation of the infection cause many diseases such as skin problems, diarrhea, weakness, loss of appetite, pallor, hair loss, edema, slowdown in growth, pallor, and decreased resistance to infection (Kartal et al., 2008). Copper toxicity occurs when this element is taken in large amounts (Neyzi & Ertugrul, 2010).

Copper is an important trace element for humans. However, excessive amounts in metabolism are toxic. Wilson's disease is seen as a result of the accumulation of this element in the liver, brain and eye. In metabolism, this disease leads to diseases such as mild and fatty appearance in the liver, cirrhosis and acute hepatitis (Teckman & Perlmutter, 1999).

3.4. Manganese (Mn)

It is an important trace element for humans. It is a trace element commonly found in plant and animal cells. It is involved in many important metabolic activities. Studies have reported that the human body contains between 10 and 40 mg of manganese (Baysal, 2002; Tosun, 2009).

Manganese is found in walnuts and nuts, soy, spinach, tea leaves, whole wheat flour and cereal seeds. It is required to be taken in the amount of 2-5 μg/day for metabolism. It is especially involved in protein and carbohydrate metabolism and the synthesis of fatty acids. It has important effects on some organs such as the respiratory system, plasma, brain and liver (Insel et al., 2006).

Particularly, it plays a role in lipid metabolism, reproduction, support and bone tissue formation, and carbohydrate metabolism. Enzymes such as mitochondial manganese, pyruvate carboxylase, superoxide dismutase and arginase contain manganese element in their structure. In addition, the manganese element activates enzymes such as transferases, hydrolases, decarboxylases and kinases (Keen et al., 1996). It is an antioxidant. In addition, it has important duties in blood coagulation, nervous system functions, infertility and increasing sexual power (Leach & Harris, 1997).

In manganese element deficiency; children and infants suffer from growth retardation, weight loss, memory problems and fatigue (Baysal, 2002; Tosun, 2009).

3.5. Iodine (I)

Iodine is a black, solid and active element. It is among a very rare element in the earth's crust. The atomic weight of the element iodine is 126.9 and it is located in group VII A of the periodic table. Iodine is a trace element that is required for thyroid hormone synthesis and taken from outside with water and nutrients (Baysal, 2004).

The daily intake of this element may vary depending on the age of the metabolism, its physiological needs and some diseases (Erdogan, 2003). Iodine is a trace element that is very rare in the human body, and its total amount in the body is around 15-20 mg. In addition, it is the main element in the synthesis of thyroid hormone and many diseases occur as a result of its deficiency (Gruters et al., 2003).

The most important cause of goiter is iodine deficiency (Brown, 2001). In addition, the deficiency of this element affects different age groups such as fetuses, newborns, adolescents and adults at different levels. These disorders, especially in metabolism, cause many diseases related to growth and mental development (Huang, 2007).

3.6. Molybdenum (Mo)

The element Mo is a powder with a dark gray and black colour, an atomic weight of 95.94, and a combustible property. This element is in group VI B of the periodic table. There are 5 different oxidation forms of molybdenum (2-6). Especially Molybdenum (IV and VI) two oxidation forms are more dominant (Barceloux, 1999).

There is molybdenum element in the active site of three enzymes (xanthine oxidase, aldehyde oxidase and sulfide oxidase) that play a role in the detoxification of the organism and/or the formation of important intermediates. In particular, deficiency in xanthine oxidase and aldehyde oxidase enzymes does not lead to any clinical results; however, deficiencies in sulfite oxidase activity have negative effects on the nervous system and organism during pre- or postnatal development (Neve, 1991).

3.7. Selenium (Se)

Selenium is a trace element in group 16 of the periodic table, atomic number 34 and atomic weight 78.96. This element is widely found in nature. It is an essential element for human and animal metabolism (Zeng & Combs, 2008).

Selenium has four different oxidation forms. These are selenide (-2), elemental selenium (0), selenite (+4) and selenate (+6) (Motchnik & Tappel, 1990). This element has important functions in metabolism (Tajaddini et al., 2015). In particular, it acts as a cofactor for many enzymes. It also takes part in tasks such as thyroid hormone mechanism, immune system regulation and antioxidant enzyme defense (Kangalgil & Yardimci, 2017).

In recent studies, selenium deficiency; it has been reported to cause various diseases such as immune system disorders, aging, cancer, cardiovascular and neurodegenerative diseases, insulin resistance, and diabetes (Wang et al., 2016). In particular, selenium is involved in the structure of the glutathione peroxidase (GSH-Px) enzyme. The GSH-Px enzyme protects the human body against cancer by preventing the proliferation of hydrogen peroxide and organic peroxides in the cells. It also has other duties such as reducing the effects of harmful substances in metabolism and taking part in reproductive health (Tosun, 2009).

3.8. Chromium (Cr)

Chromium is a metallic element in group VI B with atomic number 24 and mass number 52.01. It can exist in 0, +2, +3 and +6 oxidation states. The most important and most common form of chromium in biological systems is trivalent (Cr^{+3}). While Cr^{+6} has a toxic effect on living things, Cr^{+2} compounds are unstable compounds that are easily oxidized (Vincent, 2007).

Chromium is an essential trace element for humans and animals. Chromium, an essential trace element; it is reported that it takes part in very important events such as fat and protein synthesis, cholesterol and energy metabolism. Chromium is necessary for the biological activity of insulin and glucose tolerance may be impaired in its deficiency. Chromium is an essential element that plays a role especially in lipid and carbohydrate metabolism. At the same time, chromium reduces the levels of reactive oxygen products by showing an antioxidant effect in cases of increased oxidative stress.

Chromium deficiency in the body is manifested by increased glucose, insulin, triglyceride and LDL cholesterol levels and decreased HDL levels (Jomova & Valko, 2011). In the deficiency of this element; disorders such as lipid metabolism disorders, fasting hyperglycemia and impaired glucose tolerance occur (Goldhaber, 2003). In addition, it has been reported that Cr deficiency in metabolism increases serum glucose, lactate and triglyceride levels (Uyanık et al., 2005). It has been reported that chromium is a factor that increases immune functions and therefore resistance to infectious diseases by increasing blood cortisol levels (Vincent, 2007).

3.9. Cobalt (Co)

There is around 1.1 mg of cobalt in human metabolism. 43% is located in the muscles, 13% in the bone, and the rest in other tissues. In addition, 4% of vitamin B12 consists of cobalt element. It is known that cobalt element has a role in hypertension, use of iron, metabolism of 24 amino acids with sulfur and synthesis of thyroid hormone.

Cobalt taken in excessive amounts has a poisoning effect (Baysal, 2002). Cobalt element is found in human vitamin B12. This vitamin is necessary for the maturation of red blood cells. In addition, cobalt participates in the composition of vitamin B12 (Cobolominin), which is an antianemic liver factor. Regeneration of red blood cells cannot be achieved in cobalt (+2) deficiency. This phenomenon is called malignant anemia (Breusch & Ulusoy, 1984).

3.10. Fluorine (F)

Fluorine, one of the trace (trace) elements of metabolism, is a light greenish yellowish coloured gas. It is an important element in terms of metabolism. This element plays an important role in the development of bones and teeth, reducing bacterial enzyme activation, preventing mineral loss in tooth enamel, and cellular activities (Kalaycıoglu et al., 2000).

Studies have shown that fluoride prevents dental caries, especially in children and adults, in terms of human health. The element fluoride contributes to the restructuring of the enamel layer in the teeth. It also prevents acid formation caused by bacteria in the mouth (Ergin & Eden, 2017). It is reported that the daily amount of fluoride taken into the body with food is about 0.4 mg. Especially the amount of fluoride taken into the body with drinking water is important (Polat, 2009).

Very high concentrations of the element fluorine lead to an increase in cancer cases (Meenakshi et al., 2004). In addition, high amounts of fluoride intake cause toxic effects on metabolism and cause fluorine poisoning. In fluorine poisoning; deformities in bones and joints, bending of long bones, irreversible colour changes and disorders in teeth, tooth loss and loss of appetite have been reported (Balkaya & Açıkgöz, 2004).

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